A Goal-Oriented, Sequential, Inverse Design Method for the Horizontal Integration of a Multistage Hot Rod Rolling System

The steel manufacturing process is characterized by the requirement of expeditious development of high quality products at low cost through the effective use of available resources. Identifying solutions that meet the conflicting commercially imperative goals of such process chains is hard using traditional search techniques. The complexity in such a problem increases due to the presence of a large number of design variables, constraints and bounds, conflicting goals and the complex sequential relationships of the different stages of manufacturing. A classic example of such a manufacturing problem is the design of a rolling system for manufacturing a steel rod. This is a sequential process in which information flows from first rolling stage/pass to the last rolling pass and the decisions made at first pass influence the decisions that are made at the later passes. In this context, we define horizontal integration as the facilitation of information flow from one stage to another thereby establishing the integration of manufacturing stages to realize the end product. In this paper, we present an inverse design method based on well-established empirical models and response surface models developed through simulation experiments (finite-element based) along with the compromise decision support problem (cDSP) construct to support integrated information flow across different stages of a multistage hot rod rolling system. The method is goal-oriented because the design decisions are first made based on the end requirements identified for the process at the last rolling pass and these decisions are then passed to the preceding rolling passes following the sequential order in an inverse manner to design the entire rolling process chain to achieve the horizontal integration of stages. We illustrate the efficacy of the method by carrying out the design of a multistage rolling system. We formulate the cDSP for the second and fourth pass of a four pass rolling chain. The stages are designed by sequentially passing the design information obtained after exercising the cDSP for the last pass for different scenarios and identifying the best combination of design variables that satisfies the conflicting goals. The cDSP for the second pass helps in integrated information flow from fourth to first pass and in meeting specified goals imposed by the fourth and third passes. The end goals identified for this problem for the fourth pass are minimization of ovality (quality) of rod, maximization of throughput (productivity), and minimization of rolling load (performance and cost). The method can be instantiated for other multistage manufacturing processes such as steel making process chains hard using traditional search techniques.

1 Frame of Reference

Steel mills are involved in the production of semiproducts such as sheets or rods with certain grades of steel. Process designers are very much aware of the operating constraints and process requirements for each of the operations as they are involved in the
whole process day-in and day-out. Due to the advancements in material technology, new improved materials with enhanced properties are introduced to market posing a serious challenge to steel manufacturers. Suppose, that owing to the changing properties and performance requirements, manufacturers must produce a semiproduct such as a rod with a newer grade of steel. This new steel grade has been used at laboratory scale to produce a rod, but the challenge posed to a steel manufacturer is to scale-up production. This requires the exploration of the design set points for each unit operation in the plant scale production of the rod [1]. Plant trials are one way of achieving this, which usually takes a lot of time and are expensive. Another option is to use computational models for exploring the design set points for these operations and thereby reduce the time and cost. However, these models are for specific phenomena that occur during an integrated process. Isolated models for individual processes will not give a true representation of the whole system and the desired solution. In this context, we define horizontal integration of processes as the facilitation of information flow from one process stage to another thereby establishing the integration of manufacturing stages to realize an end product. For exploring the design set points to achieve an end product, knowledge of the operation constraints and requirements are necessary for the newer grades of steel. However, this information is not available. Therefore, the first task is to identify operating constraints and design information for each unit operation. These operating constraints are imposed by the previous and subsequent unit operations as each process step is connected and information flows from one operation to another. Such process design problems are characterized by their complexity due to the large number of variables and their relationships at multiple stages. Two types of associations are possible for such problems—sequential and nonsequential. In the case of nonsequential association, there is no definite order among the subsystems and most network problems falls under this category [2].

In this paper, we focus on demonstrating a method for designing a multistage hot rod rolling system for manufacturing a rod which is one of the semiprodutcs in a steel manufacturing process chain. We view design as a decision making process and believe that the fundamental role of a human designer is to make decisions. The hot rod rolling problem is sequential in which information flows from the first rolling stage/step to the last rolling pass and the decisions made at the first pass influence the decisions that must be made at later passes [2]. We carry out the design process by means of a goal-oriented method that uses well-established empirical models, response surface models along with the compromise decision support problem (cDSP) construct [3–5] to support integrated information flow across different stages of rolling process. The method is goal-oriented because the decisions are first made based on the end requirements identified for the product and the process at the last rolling stage, and these decisions are then passed to the preceding stages. Thus, the decisions at the first rolling stages are influenced by the decisions made at the last rolling stage thereby making this an inverse design scheme based on end goals. The cDSP is formulated using empirical models and the response surface models developed using simulation experiments and is then exercised for different design scenarios to explore the design space and to identify the best set of variables (design and operating set points) that meets the conflicting goals. Ternary plots are used to visualize these scenarios and to identify the appropriate feasible design space. The design of the multistaged rolling process is carried out using the set points identified. The entire goal-oriented inverse design method is generic and has the potential to be applied to design any set of manufacturing processes where there is sequential flow of information (material) in order to realize an end product with specified target goals. In Sec. 2, we describe the hot rod rolling process and the challenges associated with the modeling and design exploration of the process. In Sec. 3, we describe the compromise decision support problem (cDSP) construct which is the foundational construct for the goal-oriented design method proposed in this paper. In Sec. 4, we describe the problem in terms of the boundary defined and parameters considered in this study. The solution strategy in terms of process design scheme and the method adopted for this problem is also described in this section. In Sec. 5, we describe the empirical models and the response surface models developed. The mathematical formulation of the rod rolling problem using the cDSP construct is also presented in this section. The ternary analysis for visualizing and exploring the solution space is covered in Sec. 6. We conclude the paper with our key findings and closing remarks in Sec. 7. We showcase the design calculations in Appendix B.

2 The Hot Rod Rolling Process
Hot rod rolling is a complex, multistage manufacturing process that plays a critical role in producing specific grades of steel with specified target properties. The complexity in the process arises not only from the high working temperatures, but also because of the requirement to precisely control the process parameters to obtain the desired microstructure and properties. Due to increasing competition facing steel and aluminum manufacturers, there is an increasing need to make this process more flexible, agile, and energy efficient. Process designers must determine cost effective solutions to assist in decision making and improve efficiency. Multipass rolling systems design (RSD) is the preparation of a set of rolls that are laid in series in the right sequence for different rolling passes to achieve a desired profile [6]. RSD helps in producing workpieces with a desired work profile subject to the constraints of the mill with an acceptable quality, minimum cost, and maximum output. This is equivalent to a search problem where the design space is explored to satisfy the requirements in order to determine the required number of passes to achieve a product of the required dimensions with minimum defects by controlling design variables. This requires considering different behaviors of the material during rolling including geometrical, mechanical, thermal, thermo-mechanical, and metallurgical behaviors at multiple scales. Rolling is a multidisciplinary process involving reheating, interstand operations, mill engineering, roll pass design, metallurgical transformations, etc. [6].

The challenges associated with the design of a rolling system arise from the complex nature of the process due to the large number of process parameters, constraints, bounds, etc., the multistaged nature of the process involving handshakes, the hierarchical nature in terms of process—structure—property—performance relationships, multidisciplinarity requiring knowledge and expertise from different fields, complex relationships between stress/strain-temperature and microstructure that requires model coupling at different scales [6–16]. In this paper, we address some of these challenges by developing a design method using simulation models along with the compromise decision support problem construct and solution space exploration techniques to design the multiple stages of a rolling system ensuring information flow to support horizontal integration of stages in order to realize an end product. The complex search space is managed by framing a proper boundary for the problem formulated and will be explained in Sec. 4. Well-established empirical models along with a finite-element model developed for rolling is used to define the complex relationships. The academic and industrial collaboration involved in this work between people from mechanical, design, material science, and metallurgy domains helped to deal with the multidisciplinary nature of the problem. The decision support problem construct along with the solution space exploration techniques manage the uncertainty associated with the models and addresses a way of handling such complex problems from a systems design perspective. In Sec. 3, we describe the foundational construct for our work—the compromise decision support problem (cDSP) construct.
3 The Compromise Decision Support Problem Construct

In the model-based realization of complex systems, we have to deal with models that are typically incomplete, inaccurate, and not of equal fidelity. This brings into the design process the different types of uncertainties associated with the system, the parameters considered, the models considered, and the uncertainties due to their interactions [17]. From the decision-based design perspective, the fundamental role of a human designer is to make decisions given the uncertainties associated. In this regard, we define robust design as design that is relatively insensitive to changes. This involves achieving a desired performance for the system, while the sensitivity of the performance objectives with respect to the system variables are minimized [18]. Thus, the designer’s objective here is to find satisficing solutions that showcase good performance given the presence of uncertainties and not optimum solutions that are valid for narrow range of conditions, while performing poorly when the conditions are changed slightly. The cDSP is proposed by Mistree and coauthors for robust design with multiple goals [3,19]. The fundamental assumption here is that the models are not complete and accurate, opposed to the fundamental assumption in optimization where the models are complete and accurate, and the objective function can be modeled accurately so that the solution obtained is implementable. Hence, the cDSP construct is anchored in the robust design paradigm first proposed by Taguchi [20]. Using the cDSP construct, several solutions are identified by carrying out trade-offs among multiple conflicting goals. The obtained solutions are then evaluated by carrying out solution space exploration in order to identify the best solutions that satisfy the specific requirements identified. The cDSP is a hybrid formulation based on mathematical programming and goal programming. In goal programming, the target values for each goal are defined, and the emphasis is on achieving the target for each goal as close as possible. In cDSP, different weights are assigned to these goals and the compromised solutions obtained for different appropriate weights are explored. The generic formulation of cDSP is shown in Fig. 1.

There are four keywords in the cDSP formulation. All the information that is available for the designer to formulate the cDSP so as to make effective decisions is captured by the “given” keyword. In the cDSP, for each objective an achievement function \( A_i(X) \) is formulated and represents the achieved value of the \( i \)th objective as a function of a set of system variables, \( X \). The deviation variables, \( d_i \) and \( d_i^* \) represent the extent to which the goal target \( G_i \) is underachieved or overachieved with respect to the value of \( A_i(X) \). The information regarding the system variables and the deviation variables are embodied in the “Find” keyword. The information regarding the system constraints, variable bounds, and system goals are captured by the “Satisfy” keyword to determine the feasible design space and the aspiration space. The “Minimize” keyword embodies the objective function which is formulated as a function of the deviation variables. The overall goal of the designer using the cDSP construct is to minimize the deviation function so that the target values specified for the objectives are attained as closely as possible by identifying the best combination of design/system variables. The details regarding formulating the cDSP and the associated rules can be found in Bras and Mistree [19] and Mistree et al. [3].

The formulation and solving of the cDSP followed by exploration of solution space for any problem are carried out using a generalized four-step method as illustrated in Fig. 2 [21–23]. After having defined the problem and requirements, step 1 is to identify the theoretical and empirical models and relationships that exist for the process/problem of interest. Response surface models are developed to represent certain parameters as a function of the process variables. These response surface models are developed by carrying out simulation experiments, which could be finite-element model-based experiments, or other similar experiments depending on the problem of interest. The response surface models developed through simulation experiments along with the theoretical and empirical models and relationships available are used to formulate the cDSP for the process/problem that is under study (step 2).

In step 3, we exercise the cDSP for different design scenarios, and the results are recorded for each scenario. These scenarios are identified by assigning different weights to the goals of the cDSP formulated. The collective design results for different scenarios are visualized using ternary plots and the feasible design space that satisfies the design requirements in the best possible manner is identified (step 4). Multiple solutions that satisfy the design requirements are identified from the feasible design space. The designer makes design decisions from the set of solutions depending upon the preferences set for the problem under study. For the manufacturing problem under consideration, these identified design solutions are the design and operating set points.

In Sec. 4, we describe the goal-oriented method for carrying out sequential process design of manufacturing stages utilizing the methods.
cDSP construct and solution space exploration techniques to achieve the integrated design of the product and the processes. We use the hot rod rolling system design problem as an example to illustrate the efficacy of the method presented in this paper.

4 Problem Description and Solution Strategy

Rod quality depends on many factors starting from the material microstructure to the macrostructure. Key factors influencing quality include steel composition, segregation of alloying elements, distribution of inclusions, microstructure, and rod geometry. Ovality is one such geometrical property which is defined as the difference between the height of the rod section and the width from the center of the rod [24]. Ovality is desirable in the initial roll passes as it helps to reduce the geometry of the square billet. However, it is not desired in the end rod product as the output requirement is for a round/circular rod. Thus, there is a need to minimize/control the ovality induced at the last rolling stage. One way of minimizing ovality is to insure high contact between the workpiece and the roll. However, this requires high rolling loads, and thus minimization of ovality is possible at the expense of a high rolling load. Rolling load influences the overall functioning of the process and is representative of the overall process performance [2]. Rolling load ensures flow of material across passes. Higher rolling loads require increased rolling power requirements and can also yield deflections in the rolling system which is detrimental to the rolls themselves. This adds to the costs of the process. Hence, maintaining the rolling load within a target value in an acceptable range is necessary but conflicts with the objective of minimizing ovality. Excessive rolling load resulting in roll breakage and wear is detrimental to production efficiency as it conflicts with rolling process productivity which is expressed in terms of throughput [2]. Therefore, this is a multi-objective design problem with three objectives: minimize ovality, maximize throughput, and minimize rolling load subject to the rolling constraints.

In this process, the output of one stand is input for the next, and there are successive reductions of the billet at each rolling stand. Therefore, modeling this process demands information exchange between these stands as the intermediate product developed in one stand will affect the form, properties, and performance of the product developed at consecutive stands that follow which results in an impact on the end product. Therefore, a method to ensure the determination of the right combination of design variables to meet the constraints for each rolling pass and thereby meet the overall performance requirement is essential.

We have developed a computational method for this sequential problem that has information exchange between rolling passes and is used to identify the set points of the various rolling passes involved. For this example, we assume that there are four passes that follow a square-oval, oval-round, round-oval, and oval-round sequence moving from Pass 1 to Pass 4. The final requirement of the product of Pass 4 is to have minimum ovality, maximum throughput, and a minimum rolling load value within an acceptable range. The different sequential relationships that exist among passes define the problem. The constraints for the process include the range for rolling load, range for throughput, maximum value of rolling wear, minimum and maximum values of elongation, and spread for each pass. The cDSP for two passes—Pass 2 and Pass 4 are formulated. The cDSP for Pass 4 takes into account the end goals identified for the problem in terms of ovality, throughput, and rolling load. The cDSP for Pass 2 is developed to support information flow across passes and perform the design of other passes. The goals for the cDSP for Pass 2 are maximization of throughput (maintaining target throughput values achieved for Passes 3 and 4) and achieving a target value of rolling load within a defined range. The ovality goal is an end goal for the rod produced after Pass 4 and is not required for Pass 2 as the material is again subjected to deformation to oval shape in Pass 3 to facilitate progressive breakdown of geometry.

This goal-oriented sequential inverse design method proposed to design the rolling system will be explained using the information flow diagram shown in Fig. 3. In order to generalize the method, we are naming the four stages of rolling passes as "manufacturing stages" which are numbered from "n" to "n + 3."

![Goal oriented process design scheme for manufacturing stages with sequential flow of information](http://mechanicaldesign.asmedigitalcollection.asme.org/pdfaccess.ashx?url=/data/journals/jmdedb/935992/ on 01/19/2017 Terms of Use: http://www.asme.org/about-asme/terms-of-use)
We will be using the term “end product” for the rod developed after rolling and the term “input material” to refer the billet that comes from the continuous casting stage of the steel manufacturing process chain. The arrows that denote the flow of information needs to be followed to visualize the design process. There are four steps in the design method for designing these four manufacturing stages to optimize the end product.

**Step 1:** Formulation of cDSP for the last manufacturing stage \((n+3)\) using the information from the end product to be realized and the sequential relationship existing between stages \(n+2\) and \(n+3\)

The whole design process starts with the identification of requirements for the end product to be produced after the manufacturing stage \(n+3\) as shown in Fig. 3. In step 1, the cDSP for manufacturing stage \(n+3\) is formulated. The cDSP is formulated using the information available on manufacturing stage \(n+3\) and by incorporating the sequential relationship the stage \(n+3\) has with manufacturing stage \(n+2\). The requirements identified for both the end product and for the manufacturing stage \(n+3\) are embodied in this cDSP as goals. The requirements from manufacturing stage \(n+2\) along with the sequential relationships that exist are captured by the “Given” and “Satisfy” keywords of the cDSP formulated. The cDSP is exercised for different identified scenarios by assigning different weights for each goal and the scenarios that suit the design requirements the most are selected after carrying out solution space exploration using ternary plots. The system variables identified are basically the design and operating set points for manufacturing stage \(n+3\).

**Step 2:** Design of stages \(n+3\) and \(n+2\) using the design and operating set points identified and the information available from end product requirements.

In step 2, the design and operating set points generated for manufacturing stage \(n+3\) from step 1 are used to design the stage by carrying out design calculations to determine information. Design calculations essentially involve analysis to check the achievement of goals and using the design and operating set points generated to calculate the values of parameters of both the manufacturing stages using the sequential relationships that exist between them that was incorporated in the cDSP formulated. First, the design and operating set points are used to generate information for stage \(n+3\). The new design information generated for stage \(n+3\) has a sequential relationship with manufacturing stage \(n+2\), and hence, they are passed to carry out the design of manufacturing stage \(n+2\). Once the new design information is generated for manufacturing stage \(n+2\), they are again passed to manufacturing stage \(n+3\) to come up with the information which was unknown before. Thus, a cyclic process of information exchange is carried out at this step to generate new information for both the manufacturing stages using the design and operating set points identified in step 1. Step 2 ends once all the required design information for the problem formulated is identified.

**Step 3:** Formulation of cDSP for manufacturing stage \(n+1\) using the design information generated for stages \(n+2\) and \(n+3\); and the sequential information existing between stages \(n\) and \(n+1\); along with information on input material.

In step 3, the cDSP for manufacturing stage \(n+1\) is formulated. The design information generated for stages \(n+2\) and \(n+3\) are communicated to the cDSP for manufacturing stage \(n+1\). The “Given” keyword of this cDSP captures the design information from stages \(n+2\) and \(n+3\). Along with that sequential information related to stages \(n\) and \(n+1\), the initial conditions of input material are also captured during the formulation of this cDSP using the “Given” and “Satisfy” keywords. Specific requirements identified for manufacturing stage \(n+1\) are formulated as system goals. The cDSP formulated is exercised for different scenarios to find design and operating set points for manufacturing stage \(n+1\) that satisfies the requirements identified for the stage as well as the end requirements of product.

**Step 4:** Design of manufacturing stages \(n+1\) and \(n\) using the design and operating set points identified; the information available from input material and the information from stages \(n+2\) and \(n+3\)

In a similar fashion to step 2, the design and operating set points identified for manufacturing stage \(n+1\) are used to design the stage by carrying out design calculations. The design information generated for stage \(n+1\) is passed to design manufacturing stage \(n\) using the sequential relationships that exists. The information available from the input material is also used at this stage to carry out the design of manufacturing stage \(n\). The new design information generated for stage \(n\) is then communicated back to stage \(n+1\) to determine stage \(n+1\) information that was unknown before. The sequential information passing is carried out until the required design information for the problem formulated are identified. The design information generated for stages \(n\) and \(n+1\) are also used to carry out design calculations for stages \(n+2\) and \(n+3\) as the information from those stages are available in the cDSP formulated for stage \(n+1\). Hence, the final result obtained using this goal-oriented, sequential method is the design information for all the four stages \(n, n+1, n+2,\) and \(n+3\) in order to realize the requirements identified for the process as well as the end product.

The proposed four step method using the cDSP construct is generic and the method can be used for the design of other such unit operations where there is a sequential flow of information by identifying the design and operating set points that satisfy certain system goals and then design the entire system using these identified set points.

In Sec. 5, we describe the empirical models and theoretical models as well as the important relations that exist for the rod rolling problem under study. We also describe the response surface models that are developed as a part of the study here in this section. In Sec. 5.2, we explain the cDSP formulation for the Pass 4 (stage \(n+3\)) of the hot rod rolling problem. The cDSP for Pass 2 (stage \(n+1\)) which follows a similar pattern to that of Pass 4 will be explained in Appendix A. In Sec. 6, we explain the scenarios identified for the cDSP for Pass 4 and visualization of the scenarios using ternary plots to identify the design and operating set points.

### 5 Designing a Multipass Rolling System

The purpose of roll pass design is [25]: (a) To ensure the production of a correct profile within the permissible dimensional limits and with a good surface finish, free of surface defects, at the same time keeping the internal stress in the section being rolled to a minimum, (b) to ensure the maximum output at minimum cost, (c) to ease the working conditions of the rolling crew, and (d) to reduce roll wear to a minimum. For our hot rod rolling example problem, the design requirements are:

- Achieve a round profile by minimizing the ovality at the end of the fourth rolling pass.
- Maximize throughput while ensuring that the product quality is not reduced.
- Maintain a minimum rolling load within a specified range and ensuring that it never exceeds the maximum.
- Control the elongation and spread during the rolling process within specified limits.
- Control the entry and exit speeds of the stock within specified limits.
- Ensure that the wear on the rolls is within an acceptable limit.
- Obey the sequential relationships between the different rolling passes (in terms of geometry and workpiece profile, etc.)

First, a process model for rolling system is developed that ensures the flow of information through the sequential relationships between rolling passes as shown in Fig. 3. In Figs. 4(a) and 4(b), we represent the geometry for the oval and round passes with key dimensions of interest for the rolling problem. The entire breakdown sequence consists of two more such passes in a
It is calculated by dividing the cross sectional area $F_j$ by the maximum breadth $b_j$ of the filled section for a particular pass $j$.

Thus, the condition of constant volume during rolling is [25]

$$V_0 = F_0 l_0 = h_{in} b_0 l_0 = V_1 = F_1 l_1 = h_{in} b_1 l_1$$

$$V_n = F_n l_n = h_{in} b_n l_n$$

(4)

On dividing these relations [25]

$$\frac{h_{2m} b_2 l_2}{R_{in} b_1 l_1} = \gamma_m \beta \lambda = 1$$

(5)

where

$$\gamma_m = \frac{h_{2m}}{R_{in}} = \text{mean coefficient of draught}$$

(6)

$$\beta = \frac{b_2}{b_1} = \text{spread in rolling}$$

(7)

$$\lambda = \frac{F_1}{F_2} = \frac{h_{in} b_1}{h_{2m} b_2} = \frac{d_2}{l_1} \frac{w_2}{v_1} = \text{coefficient of elongation}$$

(8)

where $v_1$ is the entry speed during a rolling pass, and $w_2$ is the exit speed during the same pass. For round-oval rolling for rod production, an equivalent rectangle approximation (shown by ABCD in Figs. 4(a) and 4(b)) is carried out, and the geometrical parameters are identified during the design process.

5.1.2 Rod Ovality. The ovality of the final rod product is a serious concern for manufacturers. It is mainly due to: (i) geometric factors such as the incoming width and height of the workpiece, radius of the roll, and the roll gap, (ii) metallurgical parameters such as strain values, stress developed, temperature of the material during rolling, and (iii) rolling process parameter such as rolling speed [24].

The geometric factors such as incoming height ($h_{j-1}$) and width ($b_{j-1}$) of the workpiece will define the amount of elongation and spread that occurs while rolling. This helps to determine the ovality of the rod produced. The roll radius ($R_{max}$) and roll gap ($G_j$) are critical parameters defining rolling contact and output size. Both of these parameters affect the ovality induced. The temperature ($T_j$) during rolling is also critical and determines the material flow. Higher temperature favors flow and thus plays a role in defining ovality. Also the rolling speed ($N_j$, measured in rpm) affects the geometry formed.

Although these variables are known to influence the ovality during rolling, the exact relationships with respect to these variables are not available, and therefore, the simulation experiments using the finite-element (FE) based rolling model are carried out to determine models to predict ovality as a function of the variables identified. Appropriate ranges for the variables of interest are identified and a two level fractional factorial design of experiments (DoE) is carried out. The steps associated with the same are [24,26]:

Step 1: Fractional factorial design

The factors and factor levels for the simulations are depicted in Table 1.

A two-level six factor fractional factorial design is used for the DoE. The FE simulations are carried out using the experimental design for the different runs of DoE. The coupled temperature-
displacement finite-element model developed for the fourth oval to round rolling pass in ABAQUS is shown in Figs. 5 and 6. The material being rolled is modeled as a deformable body of oval shape and is meshed using C3D8RT, an eight-node thermally coupled brick element. The material properties, for example, conductivity as a function of temperature, elastic properties, etc., for steel is assigned to the billet. The plastic behavior of the material is described by assigning yield stress values for steel at different plastic strains. The rollers with round grooves are modeled as a discrete rigid body and are meshed using R3D4 elements. The surface profile of the rolls are modeled using the analytical models developed by Lee et al. [27]. The oval shaped billet is constrained to move along the rolling direction. The rollers are constrained to only rotate along the axis of rotation. An initial temperature is input to the billet before rolling which serves as the temperature for rolling. A surface-to-surface contact is defined between the billet surface and the grooves of the rollers. The kinematic contact method is selected for mechanical constraint formulation. The heat transfer coefficient is defined between roll gap and to air and the reported values from the literature are selected [28]. The coefficient of friction value is set to 0.3 for the rolling simulations to develop the response surfaces for ovality. In preliminary studies, the coefficient of friction was shown to have a negligible effect on ovality, however, it does have an effect on roll wear as discussed in Sec. 5.1.5. The heat due to plastic deformation value of 0.9 is used [29]. The angular velocity of roll is applied based on the average strain rate associated with the rolling pass schedule [30].

The developed FE model is validated for temperature predictions at billet center and surface, stresses developed and geometry such as the final area of the rod produced following a similar pattern as in our previous works [31,32]. The value of ovality in the rods is measured for each run and is recorded from the FE results as the absolute difference between the height and width of rod section from the center.

**Step 2: Model fitting**

In step 2, we develop response surface models for ovality by fitting the results obtained with a second-order polynomial. We carry out ANOVA and find that the effect of roll radius is negligible by analyzing the p-values obtained and thus, the roll radius is eliminated from the list of factors. The parameters of the second-order polynomial are determined using least squares regression analysis by fitting FE responses to input data. More detailed descriptions of RSM techniques and tools can be found in Myers and Montgomery [33] and Simpson et al. [34]. The response surface model thus developed for ovality with a $R^2$ value of 0.99 is

$$O_{ij} = 8.6153 \times G_j + 27.539 \times b_{j-1w} - 0.0009 \times N_j + 0.0001 \times h_{j-1} \times T_j - 0.0023 \times h_{j-1} \times N_j - 0.0041 \times G_j \times T_j - 0.0269 \times G_j \times N_j - 0.0216 \times b_{j-1w} \times T_j - 0.0026 \times b_{j-1w} \times N_j$$  (9)

The response surface of ovality model as a function of height and width of incoming workpiece with fixed values of other variables is shown in Fig. 7(a). In Fig. 7(b), we show the response of ovality model as a function of roll gap and roll rpm.

5.1.3 Throughput. Throughput defines process productivity. Throughput is expressed as a function of exit speed during rolling ($w_f$) and the final stock cross-sectional area ($F_j$) that leaves the roll [25]. The subscript $j$ refers to pass number.

$$T_{pj} = F_jw_j$$  (10)

where $F_j$ is the area of cross section for the round pass is

$$F_j = \left(\frac{bh_j^2}{4}\right)$$  (11)

where $h_j = d_j = \text{rod diameter of rod as shown in Fig. 4(b)}$.

For an oval section with a defined $(b/h)$ ratio, the cross section area is [25]

$$F_j = \left(\frac{bh_j^2}{4.35}\right)$$  (12)

where $b/h$ is a ratio defined for pass $j$. The equation is based on the values obtained from a nomogram for determining $b_{m}/h_{max}$ for common ovals relative to $s$ (roll clearance), $h$, and $b$ [25]. The expression for $b_{m}$ is [25]

$$b_{m} = (b/h)h_j$$  (13)

Hence from Eqs. (13) and (14)

$$b_{m} = \sqrt{4.35F_j}$$  (14)
5.1.4 Rolling Load. Excessive rolling load in various passes can affect the productivity, while minimum ovality is achieved through high contact and higher loads. Shinokira and Takai [35] introduced a method for calculating the effective roll radius, the projected contact area, the nondimensional roll force, and the torque arm coefficient expressed as simple functions of the geometry of the deformation zone. The rolling load \( P \) is defined as a function of a multiplier \( Q_s \), projected contact area \( (F_p) \), and mean flow strength of material \( (2k) \).

\[
P = Q_s F_p/2k
\]  

(15)

The mean flow strength of material \( (2k) \) in the pass is approximated as the yield stress of material under plain compression as expressed in Sim’s model [36]. The projected contact area is given by [37]

\[
F_p = \frac{2}{\pi} (0.9h_j)L_d = 0.573b_j L_d
\]  

(16)

where \( h_j \) is the final width after a pass. The projected length of contact in the deformation zone is [30]

\[
L_d = \sqrt{\left( R_{\text{max}} - \frac{h_o - G}{2}\right) (h_i - h_o)}
\]  

(17)

where \( R_{\text{max}} \) is the radius of the roll, \( G \) is the roll gap, and \( h_i \) and \( h_o \) are the height of the incoming and outgoing workpiece, respectively. Since there is a 90° rotation from an oval to a round pass, the incoming height of the workpiece for a round pass will be the width from the oval pass that precedes it. For a typical round pass \( j \), the formula becomes

\[
L_d = \sqrt{\left( R_{\text{max},j} - \frac{h_i - G}{2}\right) (h_{i-1} - h_i)}
\]  

(18)

The multiplier \( Q_s \) is given by [37]

\[
Q_s = -0.731 + 0.771M + \frac{1.61}{M}
\]  

(19)

where \( M \) depends on the projected contact area, \( F_p \) and the initial and final cross sections, \( F_i \) and \( F_o \), respectively [37]

\[
M = \frac{2F_p}{F_i + F_o}
\]  

(20)

For a typical pass \( j \), \( F_o = F_j \) and, \( F_i = F_{j-1} \).

5.1.5 Roll Wear During Rolling. Reducing the wear during rolling is important. To estimate it, we use an expression that estimates the change in the radius of a work roll due to wear during rolling [15]. Roll wear is expressed as [38]

\[
\Delta R_l = \frac{K \mu_j L_j^2 \sigma \exp \left[ \frac{\mu_j L_d}{h_j (2 - r)} \right]}{D^2 \sigma_{\text{roll}}}
\]  

(21)

where \( \Delta R \) is the change in roll radius, \( L_j \) is the rolled length, \( K \) is the wear constant, \( \mu_j \) is the coefficient of friction, \( L_d \) is the projected contact length, \( r \) is the reduction during rolling, \( \sigma \) is the flow strength of the material rolled, \( \sigma_{\text{roll}} \) is the flow strength of the roll, and \( D \) is the roll diameter. Here, we use a \( K = 8 \times 10^{-5} \), \( \sigma = 250 \text{ MPa} \) for the material rolled and \( \sigma_{\text{roll}} = 600 \text{ MPa} \). [15,38].

The rolled length \( L_j \) is

\[
L_j = L_j \times L_{j-1}
\]  

(22)

The value of \( L_{j-1} \) is assumed to be 3 m. The coefficient of friction, \( \mu_j \), is a system variable in this study and is between 0.3 and 0.45.

5.2 The cDSP for Roll Pass 4 (Step 1 of Method Proposed). In this section, we describe the mathematical formulation of the compromise decision support problem (cDSP) for Pass 4 of rod rolling. The cDSP for Pass 4 incorporates the end requirements identified for the rolling process. The cDSP is:

**Given:**

1. End requirements identified for the rod rolling process
   - Minimize ovality
   - Maximize throughput
   - Minimize rolling load
   - Minimum limit of rolling load, \( P_{\text{min}} = 28 \text{ ton (metric)} \)
   - Minimum limit of rolling load, \( P_{\text{max}} = 35 \text{ ton (metric)} \)
   - Maximum limit of throughput, \( T_{\text{max}} = 0.0001 \text{ m}^3/\text{s} \)
   - Maximum limit of throughput, \( T_{\text{max}} = 0.0008 \text{ m}^3/\text{s} \)
   - Target value for ovality, \( O_{\text{Target}} = 0.001 \pm 0.001 \text{ m} \)
   - Target value for rolling load, \( P_{\text{Target}} = 28 \text{ ton} \)
   - Target value for throughput, \( T_{\text{max}} = 0.0006 \text{ m}^3/\text{s} \)

2. Number of passes = 4
3. Initial billet size = 42 × 42 mm
4. Pass sequence = Square-oval-round-oval-round
5. Other parameter values for passes
6. The RSMs and well-established empirical and theoretical correlations for the oval to round pass
• Area of round section obtained after Pass 4
\[ F_4 = \left( \frac{\pi b_4^2}{4} \right) \] (23)
• Coefficient of elongation for Pass 4
\[ \lambda_4 = \frac{F_3}{F_4} \] (24)
• The theoretical width of oval Pass 3
\[ b_{3w} = \sqrt{4.35F_4} \] (25)
• The height of oval Pass 3 for a defined \( \frac{b}{h} \) ratio
\[ h_3 = \frac{b_{3w}}{\left( \frac{b}{h} \right)} \] (26)
• The width of round Pass 4 for a defined spread \( \beta_4 \)
\[ b_4 = \beta_4 h_3 \] (27)
• Radius of the curvature of oval pass
\[ R_5^* = \frac{b_{3w}^2 + h_3^2}{4h_3} \] (28)
• Mean height of the round rod produced after Pass 4
\[ h_{4in} = \frac{F_4}{b_4} \] (29)
• Theoretical diameter of roll for Pass 4
\[ D_{4o} = 2 \left( R_{\text{max,4}} + \frac{G_4}{2} \right) \] (30)
• Effective diameter of roll for Pass 4
\[ D_{4e} = D_{4o} - h_{4in} \] (31)
• Entry speed of material for Pass 4
\[ v_4 = \frac{w_4}{\lambda_4} \] (32)
• Exit speed for material for Pass 3
\[ w_3 = v_4 \] (33)
• Expression for ovality
\[ O_{4a} = 8.6153G_4 + 27.539b_{3w} - 0.0009N_4 + 0.0001h_3T_4 - 0.0023h_3N_4 - 0.0041G_4T_4 - 0.0269G_4N_4 - 0.0216b_{3w}T_4 - 0.0026b_{3w}N_4 \] (34)
• Throughput for Pass 4
\[ T_{\text{p4}} = F_4 \times w_4 \] (35)
• Rolling load in Pass 4
\[ P_4 = Q_4F_4(2k) \] (36)

(7) Variability in system variables
The system variables and their ranges are provided in Table 2.

Find:
System Variables
\( X_1 \), diameter of rod after Pass 4 \( \left( b_4 \right) \)
\( \lambda_4 \), the coefficient of elongation for Pass 4
\( v_4 \), the spread occurring in Pass 4 \( \left( b_4 \right) \)
\( w_4 \), the exit velocity for Pass 4 \( \left( w_4 \right) \)

Table 2 System variables and ranges for cDSP

<table>
<thead>
<tr>
<th>Sr. No</th>
<th>Variables</th>
<th>Ranges</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( X_1 ), diameter of rod after Pass 4 ( b_4 )</td>
<td>0.025–0.03 m</td>
</tr>
<tr>
<td>2</td>
<td>( \lambda_4 ), the coefficient of elongation for Pass 4</td>
<td>1–3</td>
</tr>
<tr>
<td>3</td>
<td>( v_4 ), the spread occurring in Pass 4 ( b_4 )</td>
<td>1–2</td>
</tr>
<tr>
<td>4</td>
<td>( w_4 ), the exit velocity for Pass 4 ( w_4 )</td>
<td>0.5–5 m/s</td>
</tr>
<tr>
<td>5</td>
<td>( X_5 ), the maximum radius of roll in Pass 4 ( R_{\text{max,4}} )</td>
<td>0.155–0.2 m</td>
</tr>
<tr>
<td>6</td>
<td>( X_{4e} ), the roll rpm in Pass 4 ( (N_4) )</td>
<td>10–20 rpm</td>
</tr>
<tr>
<td>7</td>
<td>( X_7 ), the temperature during rolling ( (T_4) )</td>
<td>1270–1280 K</td>
</tr>
<tr>
<td>8</td>
<td>( X_8 ), the roll gap ( (G_4) )</td>
<td>0.0035–0.0055 m</td>
</tr>
<tr>
<td>9</td>
<td>( X_9 ), the coefficient of friction ( (\mu_4) )</td>
<td>0.3–0.45</td>
</tr>
</tbody>
</table>

\( X_5 \), the maximum radius of roll in Pass 4 \( (R_{\text{max,4}}) \)
\( X_{4e} \), the roll rpm in Pass 4 \( (N_4) \)
\( X_7 \), the temperature during rolling \( (T_4) \)
\( X_8 \), the roll gap \( (G_4) \)
\( X_9 \), the coefficient of friction \( (\mu_4) \)

Deviation Variables
\( d_i^- \), \( d_i^+ \), \( i = 1,2,3 \)

Satisfy:
System Constraints
• Minimum coefficient of elongation constraint
\[ \lambda_4(X_2) - 1.2 \geq 0 \] (37)
• Maximum coefficient of elongation constraint
\[ 2 - \lambda_4(X_2) \geq 0 \] (38)
• Minimum spread constraint
\[ \beta_4(X_3) - 1.1 \geq 0 \] (39)
• Maximum spread constraint
\[ 1.7 - \beta_4(X_3) \geq 0 \] (40)
• Exit speed constraint
\[ w_4 - v_4(X_3) \geq 0 \] (41)
• Minimum load constraint
\[ P(X_4) - P_{\text{min}} \geq 0 \] (42)
• Maximum load constraint
\[ P_{\text{max}} - P(X_4) \geq 0 \] (43)
• Maximum wear constraint
\[ 0.0001 - \Delta R(X_4) \geq 0 \] (44)

System Goals

Goal 1: Minimize Ovality
\[ \frac{O_{\text{e,Target}}}{O_e(X_4)} - d_1^+ + d_1^- = 1 \] (45)

Goal 2: Maximize Throughput
\[ \frac{T_p(X_4)}{T_{p,\text{Target}}} + d_2^- - d_2^+ = 1 \] (46)

Goal 3: Minimize Rolling Load
\[ \frac{P_{\text{Target}}}{P(X_4)} - d_3^+ + d_3^- = 1 \] (47)
Scenarios 1–3 are for a situation where the designer wants to achieve the target of one of the goals, minimizing ovality, maximizing throughput, or minimizing rolling load. For example, in scenario 1 the preference is only for achieving the ovality goal. The region marked in dark blue contour within the red dashed lines predicts the value of this goal is 28 ton. On analyzing Fig. 10, we see that the values in the region demarcated by the blue dashed line is an ovality value very close to the specified target value is achievable. Also higher weights are assigned to the ovality goal, i.e., as the weight tends to 1, we approach the target value as closely as possible.

For the goal 2, the process designer is interested in maximizing throughput, and the target value identified is 0.0006 m³/s. In Fig. 9, we see that the values in the region demarcated by the blue dashed line achieves the target.

For goal 3, the interest of the process designer is to achieve the minimum rolling load within the defined limits. The target value for this goal is 28 ton. On analyzing Fig. 10, we see that the dark blue contour within the red dashed lines predicts the value of the goal close to the target.

Now, since the designer is interested in identifying regions that satisfy all the three goals mentioned above, there is a need to visualize these design spaces together in a single ternary plot. Therefore, we superimpose plots. The superimposed plot of the regions of interest in a ternary space is shown in Fig. 11.

In a superimposed plot, all the identified regions of interest for the three goals are merged in order to identify a single region that is common for the all the goals, if it exists. If not, the designer needs to make trade-offs among the goals. The region marked in dark blue contour within the red dashed lines predicts the value of the goal close to the target.

In Sec. 6, we exercise the cDSP formulated for different design scenarios by changing the weights associated with the deviation variables of each goal. The results for each of these scenarios are used to construct ternary plots to help a designer visualize and explore the solution space and identify design and operating set points for the rolling passes to meet the identified end requirements of the process. A similar cDSP for Pass 2 is formulated with only two goals, i.e., minimizing rolling load and achieving target throughput.

### Table 3 Scenarios with weights for goals

<table>
<thead>
<tr>
<th>Scenarios</th>
<th>( W_1 )</th>
<th>( W_2 )</th>
<th>( W_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>0.5</td>
<td>0.5</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>0.5</td>
<td>0</td>
<td>0.5</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>7</td>
<td>0.25</td>
<td>0.75</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>0.25</td>
<td>0</td>
<td>0.75</td>
</tr>
<tr>
<td>9</td>
<td>0.75</td>
<td>0</td>
<td>0.25</td>
</tr>
<tr>
<td>10</td>
<td>0.75</td>
<td>0.25</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>0</td>
<td>0.75</td>
<td>0.25</td>
</tr>
<tr>
<td>12</td>
<td>0</td>
<td>0.25</td>
<td>0.75</td>
</tr>
<tr>
<td>13</td>
<td>0.33</td>
<td>0.34</td>
<td>0.33</td>
</tr>
<tr>
<td>14</td>
<td>0.2</td>
<td>0.2</td>
<td>0.6</td>
</tr>
<tr>
<td>15</td>
<td>0.4</td>
<td>0.2</td>
<td>0.4</td>
</tr>
<tr>
<td>16</td>
<td>0.2</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>17</td>
<td>0.6</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>18</td>
<td>0.4</td>
<td>0.4</td>
<td>0.2</td>
</tr>
<tr>
<td>19</td>
<td>0.2</td>
<td>0.6</td>
<td>0.2</td>
</tr>
</tbody>
</table>
light green satisfies the requirements for ovality and throughput, while the blue region satisfies the requirements of rolling load and ovality. There is no common region that satisfies all the three goals simultaneously. The designer can either choose solutions from the regions identified or reformulate the constraints/goals to identify feasible spaces.

In this paper, we illustrate the utility of ternary plots to reformulate a problem according to new requirements and carry out solution space exploration to support decision making. For the problem under consideration, ovality goal is an important goal and cannot be relaxed at all. The goals on throughput and rolling load, however, can be relaxed. This is because of the fact that we view quality of the end product as a greater concern than productivity given that the performance criteria are met. Hence, we relax the goal on throughput even if its level drops to 0.0005 m$^3$/s. This new region of interest is identified by the blue dashed line in Fig. 12. Any combination of weights on goals in this identified region supports a throughput value greater than or equal to 0.0005 m$^3$/s.

We need to achieve minimum rolling load within the lower and upper bounds defined. Since the goal of achieving a minimum of 28 ton is not possible unless compromises are made on other goals, we are relaxing the rolling load value to 32 ton which is within the identified bounds. The acceptable new region in the ternary plot is identified by the red dashed line in Fig. 13. Any combination of weights of goals in this identified region supports a rolling load value that is less than or equal to 32 ton. We superimpose the new regions along with the region identified for minimizing ovality (Fig. 8) to see if there is a common region that satisfies all three goals for the new design preferences, Fig. 14.

In the superimposed plot for the newly identified goals, the light yellow region with multiple solutions within it denoted by the letters A to G satisfies all the newly identified goals. After exploring and analyzing each solution the designer can choose combinations from this region that meets requirements. Scenario 13 in Table 3 for which we have equal priority to the three goals ($W_1 = 0.33$, $W_2 = 0.34$, and $W_3 = 0.33$) ; point G in Fig. 14) satisfies the three goals as closely as possible compared to the other solutions within the region; therefore, this scenario and the weights associated with it is the best combination. Thus, a designer is able to identify those weight combinations that when used in the cDSP formulation helps in predicting the design set points that satisfies the conflicting goals identified. The ternary plots thus are effective tools empowering the designer to make changes in design preferences according to the demands of the problem. The designer can then analyze and explore the new scenarios in order to make effective design decisions by identifying multiple possible solutions.

Next, we identify the system variable values for Scenario 13 obtained by solving the cDSP. These system variable values are presented in Table 4. We use these system variable values to...
design Pass 4 followed by Pass 3 by using the process design scheme described in Sec. 4 and illustrated in Fig. 3. This is followed by formulating and solving the cDSP for Pass 2. The system variable values obtained by solving the cDSP and carrying out solution space exploration for Pass 2 are presented in Table 5. The design of Passes 1, 2, and 3 are carried out using the results from the Pass 2 cDSP.

### Table 4 cDSP results for Pass 4

<table>
<thead>
<tr>
<th>System variables for Pass 4 cDSP</th>
<th>Values obtained from running cDSP for Pass 4 (S13)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_4$</td>
<td>0.0260326 m</td>
</tr>
<tr>
<td>$i_4$</td>
<td>1.3</td>
</tr>
<tr>
<td>$\beta_4$</td>
<td>1.15</td>
</tr>
<tr>
<td>$w_4$</td>
<td>1.12723 m/s</td>
</tr>
<tr>
<td>$R_{\max,4}$</td>
<td>0.155012 m</td>
</tr>
<tr>
<td>$N_4$</td>
<td>17.4642 rpm</td>
</tr>
<tr>
<td>$T_4$</td>
<td>1270 K</td>
</tr>
<tr>
<td>$G_4$</td>
<td>0.004 m</td>
</tr>
<tr>
<td>$\mu_4$</td>
<td>0.3</td>
</tr>
</tbody>
</table>

### Table 5 cDSP results for Pass 2

<table>
<thead>
<tr>
<th>System variables for Pass 2 cDSP</th>
<th>Values obtained from running cDSP for Pass 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_2$</td>
<td>0.031 m</td>
</tr>
<tr>
<td>$i_2$</td>
<td>1.3</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>1.2</td>
</tr>
<tr>
<td>$w_2$</td>
<td>0.79431 m/s</td>
</tr>
<tr>
<td>$R_{\max,2}$</td>
<td>0.155 m</td>
</tr>
<tr>
<td>$G_2$</td>
<td>0.004 m</td>
</tr>
<tr>
<td>$\mu_2$</td>
<td>0.3</td>
</tr>
</tbody>
</table>

### Table 6 Summary of key design results for all passes

<table>
<thead>
<tr>
<th>Pass No.</th>
<th>Roll stand no.</th>
<th>Dimensions (mm)</th>
<th>Cross-section $F$ (mm²)</th>
<th>$\lambda$</th>
<th>$\beta$</th>
<th>Entry speed $v$ (m/s)</th>
<th>Exit speed $w$ (m/s)</th>
<th>Effective diameter $D_e$ (mm)</th>
<th>Ovality $O_v$ (m)</th>
<th>Throughput $T_p$ (m³/s)</th>
<th>Rolling Load $P$ (t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Square 42 × 42</td>
<td>1764</td>
<td></td>
<td></td>
<td></td>
<td>0.3401</td>
<td>0.611</td>
<td>333.3</td>
<td>NA</td>
<td>0.0006</td>
<td>NA</td>
</tr>
<tr>
<td>1</td>
<td>I Oval 22 × 65.3</td>
<td>981.59</td>
<td>1.797</td>
<td>1.4</td>
<td></td>
<td>0.611</td>
<td>0.79431</td>
<td>285.1</td>
<td>NA</td>
<td>0.0005997</td>
<td>40.82</td>
</tr>
<tr>
<td>2</td>
<td>II Round Ø31</td>
<td>755.07</td>
<td>1.3</td>
<td>1.2</td>
<td></td>
<td>0.611</td>
<td>0.86678</td>
<td>296.35</td>
<td>NA</td>
<td>0.0005999</td>
<td>NA</td>
</tr>
<tr>
<td>3</td>
<td>III Oval 18.3 × 55</td>
<td>691.93</td>
<td>1.0912</td>
<td>1.5</td>
<td></td>
<td>0.79431</td>
<td>0.86678</td>
<td>288.7</td>
<td>0.001004</td>
<td>0.0005999</td>
<td>30.002007</td>
</tr>
<tr>
<td>4</td>
<td>IV Round Ø26</td>
<td>532.26</td>
<td>1.3</td>
<td>1.15</td>
<td></td>
<td>0.86678</td>
<td>1.1272</td>
<td>0.001004</td>
<td>0.0005999</td>
<td>30.002007</td>
<td>NA</td>
</tr>
</tbody>
</table>

NA: Not applicable for the formulated problem under study.
The design of all the passes by following the process design scheme is shown in Appendix B. The results of the roll pass design calculations are summarized in Table 6, and the pass dimensions are shown in Figs. 15–18.

We discuss the design results summarized in Table 6 briefly here. We achieve a round rod of diameter 26 mm at the end of Pass 4 with ovality of 0.0006 m³/s and a rolling load value of almost 30 ton. This is achieved with a coefficient of elongation of 1.3 and spread of 1.15 occurring, while the material is rolled in Pass 4. The entry speed of the material for Pass 4 is 0.866 m/s, and exit speed is 1.127 m/s. The effective roll diameter is obtained as 288.7 mm for this pass.

The design of Pass 3 results in an oval stock of dimensions 18.3 × 55 mm. To design Pass 3, the spread value is assumed to be 1.5 and the coefficient of elongation is 1.0912. The entry speed of stock is 0.7943 m/s, and the exit speed is the same as the entry speed of Pass 4. The maximum roll radius is assumed to be the same as Pass 4 and an effective roll diameter of 296.3 mm for Pass 3 is based on this assumption. The design is able to achieve/maintain a throughput of almost 0.0006 m³/s for Pass 3.

The design of Pass 2 results in a round stock with diameter of 31 mm. The coefficient of elongation and spread for this Pass are 1.3 and 1.2, respectively. The entry is 0.611 m/s, and the exit speed is the same as the entry speed of Pass 3. The effective diameter obtained for this pass is 285 mm. The target rolling load value of 40 ton for Pass 2 is achieved, and the throughput is maintained at 0.0006 m³/s.

The design of Pass 1 results in an oval stock of dimensions 22 × 65.3 mm. The coefficient of elongation for this pass is 1.797. The spread value is assumed to be 1.4. The entry speed for this pass is 0.3401 m/s. The exit speed is same as the entry speed of Pass 2. The maximum roll radius is assumed to be same as Pass 2 and Pass 4, and the effective roll diameter is 333 mm based on this assumption. The throughput value of 0.0006 m³/s is achieved with this configuration.

7 Closing Remarks

In this paper, we propose a method based on well-established empirical models and response surface models developed through simulation experiments along with the compromise decision support problem (cDSP) construct to support integrated information flow through different stages of a multistage hot rod rolling system (horizontal integration). We illustrate the efficacy of the proposed goal-oriented, sequential inverse design method using hot rod rolling as an example. Here, the design decisions are first made at the last rolling pass based on the end requirements of the process. We allow these design decisions to be passed to the preceding rolling passes by following the sequential relationships existing between the passes in an inverse manner. We carry out the design of individual passes by allowing design information to be passed back and forth between passes using the sequential relationships. The formulation of individual cDSPs for passes helps to organize the sequential information flow and provides the ability to the designer to consider specific goals associated with each rolling pass and integrating them with the end goals. The ternary analysis feature incorporated in the method provides the designer with the capability of exploring the solution space and identifying feasible regions that satisfies the different goals identified for a particular stage of the manufacturing process chain. The proposed method has the potential to be used for identifying design set points for a chain of unit operations that are connected in sequence. Once the information flow between operations and the empirical and the simulation/response surface models necessary to establish relationships are available, a designer will be able to use this method to achieve the integrated decision-based design of the product and the processes.

Acknowledgment

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Appendix A: cDSP Formulation for Pass 2

In this section, we describe the mathematical formulation of the compromise decision support problem (cDSP) for Pass 2 of rod rolling. The cDSP for Pass 2 incorporates the design information passed from Pass 3 and Pass 4. The cDSP reads as follows:

Given:

1. design information passed from Pass 3 and Pass 4
2. requirements at Pass 2

- achieve target throughput (results obtained from Pass 4 design)
- achieve target rolling load
- target value for throughput, \( T_{\text{p4,Target}} = 0.0006 \text{ m}^3/\text{s} \)
- target value for rolling load, \( P_{\text{Target}} = 40 \text{ ton} \)
- minimum value of rolling load, \( P_{\text{min}} = 35 \text{ ton} \)
- maximum value of rolling load, \( P_{\text{max}} = 45 \text{ ton} \)
- minimum value of throughput, \( T_{\text{p4,min}} = 0.0001 \text{ m}^3/\text{s} \)
- maximum value of throughput, \( T_{\text{p4,max}} = 0.0008 \text{ m}^3/\text{s} \)

3. initial billet size = 42 × 42 mm
4. other parameter values for passes
5. the regression equations and well-established empirical and theoretical correlations for the oval to round pass for Pass 2
6. variability in system variables

The ranges identified for the system variables are provided in Table 7.

Find:

System Variables

- \( X_1 \), diameter of rod after Pass 2 (\( h_2 \))

Table 7 System variables and ranges for Pass 2 cDSP

<table>
<thead>
<tr>
<th>Sr. No</th>
<th>Variables</th>
<th>Ranges</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( X_1 ), diameter of rod after Pass 2 (( h_2 ))</td>
<td>0.03–0.04 m</td>
</tr>
<tr>
<td>2</td>
<td>( X_2 ), the coefficient of elongation for Pass 2 (( i_2 ))</td>
<td>1–3</td>
</tr>
<tr>
<td>3</td>
<td>( X_3 ), the spread occurring in Pass 2 (( f_2 ))</td>
<td>1–2</td>
</tr>
<tr>
<td>4</td>
<td>( X_4 ), the exit velocity for Pass 2 (( v_2 ))</td>
<td>0.5–3 m/s</td>
</tr>
<tr>
<td>5</td>
<td>( X_5 ), the maximum radius of roll in Pass 2 (( R_{\text{max,2}} ))</td>
<td>0.155–0.2 m</td>
</tr>
<tr>
<td>6</td>
<td>( X_6 ), the roll gap (( G_2 ))</td>
<td>0.0035–0.0055 m</td>
</tr>
<tr>
<td>7</td>
<td>( X_7 ), the coefficient of friction (( \mu_2 ))</td>
<td>0.3–0.45</td>
</tr>
</tbody>
</table>
- $X_2$, the coefficient of elongation for Pass 2 ($\lambda_2$)
- $X_4$, the spread occurring in Pass 2 ($\beta_2$)
- $X_4$, the exit velocity for Pass 2 ($w_2$)
- $X_5$, the maximum radius of roll in Pass 2 ($R_{\text{max},2}$)
- $X_6$, the roll gap ($G_2$)
- $X_7$, the coefficient of friction ($\mu_2$)

Deviation Variables
- $d_{i-}$, $d_{i+}$, $i = 1, 2$

System Constraints
- Minimum coefficient of elongation constraint: $\lambda_2(X_2) - 1.2 \geq 0$
- Maximum coefficient of elongation constraint: $2 - \lambda_2(X_2) \geq 0$
- Minimum spread constraint: $\beta_2(X_3) - 1.1 \geq 0$
- Maximum spread constraint: $1.7 - \beta_2(X_3) \geq 0$
- Exit speed constraint: $w_2 - v_r(X_3) \geq 0$
- Minimum load constraint: $P(X_3) - P_{\text{min}} \geq 0$
- Maximum load constraint: $P_{\text{max}} - P(X_3) \geq 0$
- Maximum wear constraint: $0.0001 - \Delta R(X_3) \geq 0$

System Goals

Goal 1:
- Maximize Throughput:
  $$T_p(X_3) + d_{i-} - d_{i+} = 1$$

Goal 2:
- Minimize Rolling Load:
  $$\frac{P_{\text{Target}}}{P(X_3)} - d_{i-} + d_{i+} = 1$$

Variable Bounds
Defined in Table 7

Bounds on deviation variables
- $d_{i-}, d_{i+} \geq 0$ and $d_{i-} \cdot d_{i+} = 0, i = 1, 2$

Minimize:
- Minimize the deviation function
  $$Z = \sum_{i=1}^{2} W_i (d_{i-} + d_{i+}); \quad \sum_{i=1}^{2} W_i = 1$$

Appendix B: Design Calculations (Refer to Fig. 3)
In this section, we describe the design calculations carried out for each pass based on the cDSP results obtained that are showcased in Tables 4 and 5. The design process is carried out following the sequential relationships that exist between passes ensuring the flow of information pattern as shown in Fig. 3.

Step 1: Formulation of cDSP for roll Pass 4 using the information from the end product to be realized and the sequential relationship existing between roll Pass 3 and 4

The cDSP for Pass 4 is formulated in terms of the end requirements of minimizing ovality, maximizing throughput, and minimizing rolling load within the system constraints and bounds defined. The cDSP is exercised for different scenarios and ternary plots are used to identify best region, and the results are summarized in Table 4.

Step 2: Design of Passes 4 and 3 using the design and operating set points identified and the information available from end product requirements

We calculate the area of the round rod using the height value obtained for the rod from cDSP results. Cross-sectional area of material after Pass 4:

$$A_4 = \pi h_4^2 / 4 = 532.26 \text{ mm}^2$$

Entry speed of material for roll Pass 4:

$$v_4 = \frac{w_4}{l_4} = 0.8671 \text{ m/s}$$

Throughput achieved in Pass 4:

$$T_{p4} = F_4 \times w_4 = 0.0005999 \text{ m}^3 / \text{s}$$

We carry out the design calculations for Pass 3 based on the cross-sectional area of rod and elongation coefficient (cDSP result) obtained after Pass 4. We also define some requirements for Pass 3 such as meeting the throughput same as that of Pass 4.

Cross-sectional area of material after Pass 3:

$$A_3 = F_3 \times l_4 = 691.93 \text{ mm}^2$$

Theoretical width of oval pass after Pass 3:

$$b_{3w} = \sqrt{4.35 \times F_3} = 54.86 \text{ mm}$$

Height of material after Pass 3 (assuming $h/b$ ratio = 3):

$$h_3 = \frac{b_{3w}}{(b/h)} = 18.28 \text{ mm}$$

Radius of curvature of oval Pass 3:

$$R_3^* = \frac{b_{3w}^2 + h_3^2}{4h_3} = 45.72 \text{ mm}$$

Exit speed of material for roll Pass 3:

$$w_3 = v_4 = 0.8671 \text{ m/s}$$

Throughput to be maintained in Pass 3 (Given):

$$T_{p3} = T_{p4} = 0.0005999 \text{ m}^3 / \text{s}$$

We carry out design calculations for Pass 4 now with the new information generated for Pass 3:

Width of round profile (approximated rectangle) after Pass 4:

$$b_4 = h_4 \times h_4 = 31.03 \text{ mm}$$

Mean height after Pass 4:

$$h_{4m} = \frac{h_4}{b_4} = 25.31 \text{ mm}$$

Theoretical diameter of roll for Pass 4:

$$D_{r4} = 2 \left( R_{\text{max},4} + \frac{G_4}{2} \right) = 314 \text{ mm}$$

Effective diameter of roll for Pass 4:

$$D_{w4} = D_{r4} - h_{4m} = 288.7 \text{ mm}$$

Step 3: Formulation of cDSP for roll Pass 2 using the design information generated for Passes 3 and 4, and the sequential information existing between Passes 1 and 2; along with information on input material (billet)

The designer formulates the cDSP for Pass 2 after finding the results from Passes 3 and 4. For example, the range of the height of rod for Pass 2 is identified based on the dimensions achieved in Passes 3 and 4. Another example is the rolling load target value. Since there is a chance of having higher rolling load during Pass 2 due to larger stock that is being rolled than Pass 4, the target,
Next, the design calculations for Pass 1 is carried out using Pass material and the information from Passes 3 and 4 ating set points identified; the information available from input achieved in Passes 3 and 4. Thus, the designer makes judge-
ments based on the information obtained from the information as it develops.

Step 4: Design of roll Passes 2 and 1 using the design and operating set points identified; the information available from input generated followed by collecting all the results for Passes 1 and 2.

Width of round profile (approximated rectangle) after Pass 2:

\[ b_2 = \beta_2 \times h_1 = 26.14 \text{ mm} \]

Mean height after Pass 2:

\[ h_2 = \frac{F_1}{b_2} = 28.88 \text{ mm} \]

Theoretical diameter of roll for Pass 2:

\[ D_{2t} = 2 \left( R_{\max,2} + \frac{G_2}{2} \right) = 314 \text{ mm} \]

Effective diameter of roll for Pass 4:

\[ D_{w4} = D_{t4} - h_{3m} = 285.1 \text{ mm} \]

With the information generated for Passes 1 and 2, the design calculations for Passes 3 and 4 are carried out completing design results for Passes 1, 2, 3, and 4.

Coefficient of elongation for Pass 3:

\[ \lambda_3 = \frac{F_2}{F_3} = 1.091 \]

Width of oval profile (approximated rectangle) after Pass 3 (assuming \( \beta_3 = 1.5 \)):

\[ b_3 = \beta_3 \times b_2 = 39.2 \text{ mm} \]

Mean height of material after Pass 3:

\[ h_3 = \frac{F_3}{b_3} = 17.65 \text{ mm} \]

Effective diameter of roll for Pass 3 (Assuming a theoretical diameter for rolls in Pass 1, \( D_{t3} = 314 \text{ mm} \)):

\[ D_{w3} = D_{t3} - h_{3m} = 296.35 \text{ mm} \]

Entry speed of material for roll Pass 3:

\[ v_3 = \frac{w_3}{h_3} = 0.7943 \text{ m/s} \]

Exit speed of material for roll Pass 3:

\[ w_3 = v_3 \times h_3 = 0.867 \text{ m/s} \]

This completes the design of the rolling passes with the determination of all the key dimensions presented in Figs. 4(a) and 4(b).
References


