Throughput Maximization in CSMA Networks with Collisions*

Sankrith Subramanian[1], Eduardo L. Pasiliao[2], John M. Shea[1], Marco M. Carvalho[3], and Warren E. Dixon[1]

Abstract— **In the Medium Access Control (MAC) layer of a wireless network that uses Carrier Sense Multiple Access (CSMA), the performance is limited by collisions that occur because of carrier sensing delays associated with propagation and the sensing electronics. In this paper, we use a continuous-time Markov model to analyze and optimize the performance of a system using CSMA with collisions caused by sensing delays. The throughput of the network is quantified using the stationary distribution of the Markov model. An online algorithm is developed for the unconstrained throughput maximization problem. Further, a constrained problem is formulated and solved using a numerical algorithm. Simulations are provided to analyze and validate the solution to the unconstrained and constrained optimization problems.**

I. Introduction

There has been a significant effort to model various forms of CSMA protocols over the past few years [1]–[3]. Work on MAC layer throughput optimization focuses on manipulating specific parameters of the MAC layer including, for example, window sizes and transmission rates, to maximize / optimize the throughput in the presence of constraints. For example, Carrier Sense Multiple Access (CSMA) Markov chain based throughput modeling and analysis of the MAC algorithms were introduced in [1], [2], while performance and throughput analysis of the conventional Binomial exponential backoff algorithms have been investigated in [4], [5]. In most cases, previous MAC-layer optimization algorithms have focused primarily on parameters and feedback from the MAC layer by excluding collisions during the analysis (cf. [1], [3]). In this paper, we develop a continuous-time Markov model for a system using CSMA that incorporates the effect of collisions and allows optimization of the transmission rates of the network to maximize throughput or meet specified throughput targets. The purpose of this work is to develop approaches that will be useful in future cross-layer optimization and control algorithms.

Preliminary work on CSMA throughput modeling and analysis was done in [1] based on the assumption that the propagation delay between neighboring nodes is zero. A continuous Markov model was developed that provided the framework and motivation for this work. In [3], a collision-free model is used to quantify and optimize the throughput of the network. The feasibility of the arrival rate vector guarantees the reachability of maximum throughput, which in turn satisfies the constraint that the service rate is greater than or equal to the arrival rate, assuming that the propagation delay is zero.

However, in more realistic communication networks, effects of propagation delay play a crucial role in modeling and analyzing the throughput of the network. Recent efforts attempted various strategies to include delay models in the throughput model. For example, in [6], delay is introduced, and is used to analyze and characterize the achievable rate region for static CSMA schedulers. Collisions, and hence delay is incorporated in [7] in the Markov model, and the mean transmission length of the packets are used as the control variable to maximize the throughput. In this paper, a model for propagation delay is proposed and incorporated in the model for throughput. This model allows for the transmission rates to be selected to maximize throughput in an unconstrained optimization problem and to meet feasible throughput goals in a constrained optimization problem.

II. Network Model

We consider an infrastructure network, such as a wireless local area network (WLAN), consisting of an access point and n mobile stations. There are n links connecting the stations to the AP, as shown in Fig. 1. We assume that all of the nodes in the network can sense the transmissions of all of the other nodes, provided that the transmissions do not begin within a fixed sensing delay, δT_s. If two or more nodes initiate packet transmission within δT_s, there will be a collision, and all of the packets involved in the transmission are assumed to be lost. In a typical CSMA network, the transmitter of node k backs off for a random period before it sends a packet to its destination node, if the channel is idle. If the channel is busy, the transmitter freezes its backoff counter until the channel is idle again. We assume that the backoff time, or the waiting time of each link k is exponentially distributed with mean $1/R_k$. The objective in this paper is to determine the optimal values of the mean transmission rates R_k, $k = 1, 2, ..., n$, so that the throughput in the network is either maximized (if all of the nodes are assumed to have the same traffic requirements) or so that the throughput requirements of the nodes are met (if feasible). For this purpose, a Markovian model is used, and its states, defined as $x^i \in \{0,1\}^n$, represents the status of the network where 1 represents an active link, and 0 represents an idle

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link. For example, if the kth link in state i is active, then $x_k^i = 1$.

We define two sets of indices below for the collision-free transmission states, $\mathscr{A}$, and the collision states, $\mathscr{C}$:

$$\mathscr{A} = \left\{ i \mid \sum_{k=1}^{n} x_k^i = 1 \right\}$$
$$\mathscr{C} = \left\{ i \mid \sum_{k=1}^{n} x_k^i > 1 \right\}$$

$$\text{where} \quad x_k^i = \begin{cases} 1 & \text{if link } k \text{ in state } i \text{ is active,} \\ 0 & \text{otherwise.} \end{cases}$$

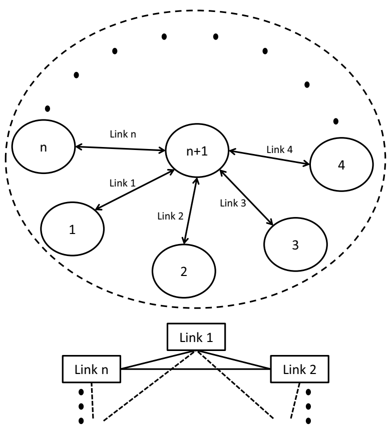

Fig. 1. A n-link network scenario and conflict graph.

Previous work in this field assumed that the propagation delay between neighboring nodes is zero (cf. [1], [3]), and hence, the motivation behind this paper is to maximize the throughput in the network in the presence of sensing delays, and consequently collisions. Although collisions due to hidden terminals are possible, this paper focuses on collisions due to sensing delay. Nevertheless, the formulations in this paper can be extended to hidden terminals as well using the formulations of the rate of transitions in [1]. The following section explains the continuous CSMA Markov chain in detail.

III. CSMA Markov Chain

Formulations of Markov models for capturing the MAC layer dynamics in CSMA networks were developed in [1], [2]. The stationary distribution of the states and the balance equations were developed and used to quantify the throughput. Recently, a continuous time CSMA Markov model without collisions was used in [3] to develop an adaptive CSMA to maximize throughput. Collisions were introduced in [7] in the Markov model, and the mean transmission length of the packets are used as the control variable to maximize the throughput. Since most applications experience random length of packets, the transmission rates (packets/unit time), R_k, $k = 1, 2, \ldots, n$, of the links are used as a practical measure in this paper.

The model for the waiting times is based on the CSMA random access protocol. The probability density function of the waiting time T_k is given by

$$f_{T_k}(t_k) = \begin{cases} R_k \exp(-R_k t_k) & \text{for } t_k \geq 0, \\ 0 & \text{for } t_k < 0. \end{cases}$$

Due to the sensing delay experienced by the nodes in the network, the probability that link k becomes active within a time duration of δT_s from the instant link l becomes active is

$$p_{c_k} \triangleq 1 - \exp\left(-R_k \delta T_s\right) \tag{1}$$

by the memoryless property of the exponential random variable. Thus, the rate of transistion G_i to one of the non-collision states in the Markov chain in Fig. 2 is defined as

$$G_i = \sum_{k=1}^{N} \left(x_k^i R_k \prod_{l \neq k} \left(1 - p_{c_l}\right)^{\left(1 - x_l^i\right)} \right) \quad \forall i \in \mathscr{A}. \tag{2}$$

The rate of transistion G_i to one of the collision states is given by

$$G_i = \sum_{k=1}^{N} \left(x_k^i R_k \prod_{l \neq k} \left(p_{c_l}\right)^{x_l^i} \left(1 - p_{c_l}\right)^{\left(1 - x_l^i\right)} \right) \quad \forall i \in \mathscr{C}. \tag{3}$$

The state $(1,1)$ in Fig. 2 represent the collision state, which occurs when a link tries to transmit within a time span of δT_s from the instant another link starts transmitting.

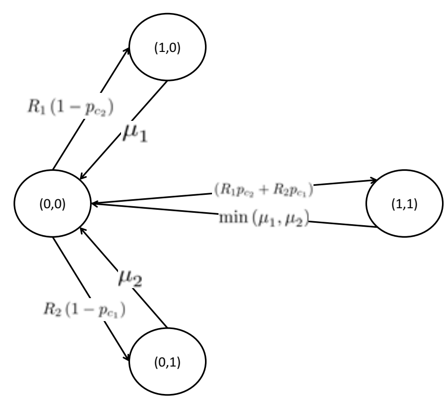

Fig. 2. CSMA Markov chain for a 2-link scenario with collision states.

The primary objective of modeling the network as a continuous CSMA Markov chain is to maximize the probability of being in the collision-free transmission states. For this purpose, the stationary distribution of the continuous time Markov chain is defined as

$$p\left(x^i\right) \triangleq \frac{\exp\left(r_i\right)}{\sum_j \exp\left(r_j\right)}, \tag{4}$$

where

$$
r_i \triangleq \begin{cases} \ln\left\{\dfrac{\sum_{k=1}^{n}\left(x_k^i R_k \prod_{l\neq k}\left(1-p_{c_l}\right)^{\left(1-x_l^i\right)}\right)}{\sum_{k=1}^{n} x_k^i \mu_k}\right\} & \text{for } i \in \mathscr{A}, \\ \dfrac{\sum_{k=1}^{n}\left(x_k^i R_k \prod_{l\neq k}\left(p_{c_l}\right)^{x_l^i}\left(1-p_{c_l}\right)^{\left(1-x_l^i\right)}\right)}{\min_{m:x_m^i\neq 0}\left(\mu_m\right)} & \text{for } i \in \mathscr{C}, \\ 1 & \text{otherwise.} \end{cases} \quad (5)
$$

where $1/\mu_i$ is the mean transmission length of the packets if the network is in one of the states in set $\mathscr{A}$. The set $\mathscr{A} \triangleq \mathscr{C}^c \setminus (0,0)^T$ represent the set of all collision-free transmission state indices, where the elements in the set $\mathscr{C}$ represent the collision state indices, and the elements in the set $\mathscr{C}^c$ represent the non-collision state indices. In (5), the definitions for the rate of transitions in (2) and (3) are used, and (4) satisfies the detailed balance equation (cf. [8]).

IV. Throughput Maximization

To quantify the throughput, a log-likelihood function is defined as the summation over all the collision-free transmission states as

$$
F(R) \triangleq \sum_{i\in\mathscr{A}} \ln\left(p\left(x^i\right)\right). \quad (6)
$$

By using the definition for $p\left(x^i\right)$ in (4), the log-likelihood function can be rewritten as

$$
\begin{aligned}
F(R) \; = \; & \sum_{k=1}^{n} \ln\left(\frac{R_k}{\mu_k}\right) - (n-1)\sum_{k=1}^{n} R_k \delta T_s \\
& - n \ln\Bigg[\sum_{k=1}^{n} \frac{R_k}{\mu_k} \prod_{l\neq k} \exp\left(-R_l \delta T_s\right) \\
& + \sum_{i\in\mathscr{C}} \exp\Bigg(\frac{1}{\min_{m:x_m^i\neq 0}\left(\mu_m\right)} \sum_{k=1}^{N}\Bigg(x_k^i R_k \\
& \prod_{l\neq k}\left(p_{c_l}\right)^{x_l^i}\left(1-p_{c_l}\right)^{\left(1-x_l^i\right)}\Bigg)\Bigg) + \exp(1)\Bigg]. \quad (7)
\end{aligned}
$$

For example, the log-likelihood function in (6) for a 2-link scenario can be expressed as

$$
\begin{aligned}
F\left(R_1, R_2\right) \; = \; & \log\left(\frac{R_1}{\mu_1}\right) - R_2 \delta T_s + \log\left(\frac{R_2}{\mu_2}\right) - R_1 \delta T_s \\
& - 2\log\Bigg[\exp\left(\log\left(\frac{R_1 \exp\left(-R_2 \delta T_s\right)}{\mu_1}\right)\right) \\
& + \exp\left(\log\left(\frac{R_2 \exp\left(-R_1 \delta T_s\right)}{\mu_2}\right)\right) \\
& + \exp\Bigg(\frac{1}{\min\left(\mu_1, \mu_2\right)} \\
& \left(R_1\left(1-\exp\left(-R_2 \delta T_s\right)\right)\right. \\
& \left.+R_2\left(1-\exp\left(-R_1 \delta T_s\right)\right)\right)\Bigg) + \exp(1)\Bigg].
\end{aligned}
$$

The convex function $F(R) \leq 0$, since $\ln\left(p\left(x^i\right)\right) \leq 0$ (cf. [9]). The optimization problem is defined as

$$
\min_R \left(-F(R)\right). \quad (8)
$$

Taking the partial derivative with respect to R_k in (7) yields

$$
\begin{aligned}
\frac{\partial F(R)}{\partial R_k} \; = \; & \frac{1}{R_k} - (n-1)\delta T_s - \frac{n}{D}\Bigg\{\frac{1}{\mu_k}\prod_{l\neq k}\exp\left(-R_l \delta T_s\right) \\
& -\left(\sum_{m:m\neq k}\frac{R_m}{\mu_m}\prod_{l\neq m,k}\exp\left(-R_l \delta T_s\right)\right) \\
& \delta T_s \exp\left(-R_k \delta T_s\right) + \sum_{x^i\in\mathscr{C}}\Bigg[\exp\Bigg(\frac{1}{\min_{m:x_m^i\neq 0}\left(\mu_m\right)} \\
& \sum_{k=1}^{N} x_k^i R_k \prod_{l\neq k}\left(p_{c_l}\right)^{x_l^i}\left(1-p_{c_l}\right)^{\left(1-x_l^i\right)}\Bigg) \\
& \Bigg(\frac{1}{\min_{m:x_m^i\neq 0}\left(\mu_m\right)}\Bigg(x_k^i \prod_{l\neq k}\left(p_{c_l}\right)^{x_l^i}\left(1-p_{c_l}\right)^{\left(1-x_l^i\right)} \\
& +\Bigg(\sum_{m:m\neq k} x_m^i R_m \prod_{l\neq m,k}\left(p_{c_l}\right)^{x_l^i}\left(1-p_{c_l}\right)^{\left(1-x_l^i\right)} \\
& \frac{\partial}{\partial R_k}\left(\left(p_{c_k}\right)^{x_k^i}\left(1-p_{c_k}\right)^{\left(1-x_k^i\right)}\right)\Bigg)\Bigg)\Bigg)\Bigg]\Bigg\}, \quad (9)
\end{aligned}
$$

$k = 1, 2, \ldots, n-1$, where

$$
\begin{aligned}
D \; \triangleq \; & \sum_{k=1}^{n}\exp\left(\ln\left(\frac{R_k \prod_{l\neq k}\exp\left(-R_l \delta T_s\right)}{\mu_k}\right)\right) \\
& + \sum_{i\in\mathscr{C}}\exp\frac{\sum_{k=1}^{N}\left(x_k^i R_k \prod_{l\neq k}\left(p_{c_l}\right)^{x_l^i}\left(1-p_{c_l}\right)^{\left(1-x_l^i\right)}\right)}{\min_{m:x_m^i\neq 0}\left(\mu_m\right)} \\
& + \exp(1). \quad (10)
\end{aligned}
$$

An online gradient-based algorithm is used to solve the problem in (8). The gradient law is defined as

$$
\ln R_k(t+T) = \ln R_k(t) + K\frac{\partial F(R)}{\partial R_k}, \quad (11)
$$

$k = 1, 2, \ldots, n-1$, where $K \in \mathbb{R}$ is the step size, T is the time interval between updates, and $\partial F(R)/\partial R_k$ is defined in (9). The calculation of $\partial F(R)/\partial R_k$ at the transmitter of link k is determined as follows. The transmitting node of link k calculates the steady-state probabilities of the states $p\left(x^i\right)$, $\forall i \in \mathscr{A}$ every T unit time. The transmitting node of link k calculates the steady-state probabilities of the collision-free transmission states alone, since these are sufficient to estimate the mean transmission rates $R_m, m \neq k$ using (4). For a n-link case, the transmitter of link k needs to solve the following set of independent nonlinear equations (after manipulations of (4)),

$$R_l \exp\left(-R_l \delta T_s\right) = \left(\frac{\Pr\left(\text{Only link } l \text{ is active}\right)}{\exp\left(-R_k \delta T_s\right)}\right) \cdot \left(\frac{R_k}{\Pr\left(\text{Only link } k \text{ is active}\right)}\right), \quad (12)$$

$\forall l \neq k$. Note that link k can use its current value of the mean transmission rate R_k to solve (12). The value of T can be chosen sufficiently large so that $p\left(x^i\right)$, $\forall i \in \mathscr{A}$ can be measured accurately. Further, large T affects identification of the collision-free transmission states by the transmitter of link k using the Carrier Sense (CS) protocol. The maximum sensing delay δT_s and the mean transmission lengths $1/\mu_k$, $k = 1, 2, ..., n-1$ are assumed to be known at all the transmitting nodes. Hence, the algorithm is *distributed*.

In addition to maximizing the log-likelihood function, certain constraints must be satisfied. The service rate $S(R)$ at each transmitter of a link needs to be equal to the arrival rate λ, and the chosen mean transmission rates R_k, $k = 1, 2, ..., n$, need to be non-negative. Thus, the optimization problem can be formulated as

$$\min_R \left(-F\left(R\right)\right)$$

subject to

$$\ln \lambda - \ln S(R) = 0, \quad (13)$$

and

$$-R \leq 0, \quad (14)$$

where $R \in \mathbb{R}^n$, $S(R) \in \mathbb{R}^{n-1}$, and $\lambda \in \mathbb{R}^{n-1}$. The service rate for a link is the rate at which a packet is transmitted, and is quantified as

$$S_k(R) \triangleq \frac{\exp\left(\ln\left(\frac{R_k \prod_{l \neq k} \exp(-R_l \delta T_s)}{\mu_k}\right)\right)}{D},$$

$k = 1, 2, ..., n-1$, and D is defined in (10). Note that $\ln \lambda_k - \ln S_k(R) = 0$, $\lambda_k > 0$ is convex for all k. The optimization problem defined above is a convex constrained nonlinear programming problem, and obtaining a analytical solution is difficult. There are numerical techiques adopted in the literature which have investigated such problems in detail [9]–[12]. In this work, a suitable numerical optimization algorithm is employed to solve the optimization problem defined in (8), (13), and (14).

The following section analyzes the numerical results obtained by solving the unconstrained problem of (8), and compares the mean transmission rates obtained online from the distributed algorithm of (11) with the optimal values. Further, numerical analysis of the constrained problem defined in (8), (13), and (14) is performed.

V. Simulation Results

A CSMA platform is developed using MATLAB that uses the standard carrier sense channel access protocol. A slot time of 10 μs is used, and the mean transmission lengths of the packets, $1/\mu_k$, $k = 1, 2, ..., n$, are set to 1 ms. An update time of $T = 100$ ms and a step size of $K = 5$ are used. The distributed algorithm in (11) is used to generate the rate updates for each transmitting node $k = 1, 2, ..., n-1$. The transmitter of link k calculates the steady-state distribution of the states $p\left(x^i\right)$, $\forall i \in \mathscr{A}$ every T unit time, and estimates the mean transmission rates of the other transmitting nodes $R_m, m \neq k$ using (12) to calculate (9). A nonlinear equation solver (MATLAB built-in function `fzero`) can be used to solve (12). The mean transmission rate updates can thus be calculated from (11). For a 3-link network with sensing delay of 0.001 ms, the mean transmission rates convergence is shown in Fig. 3.

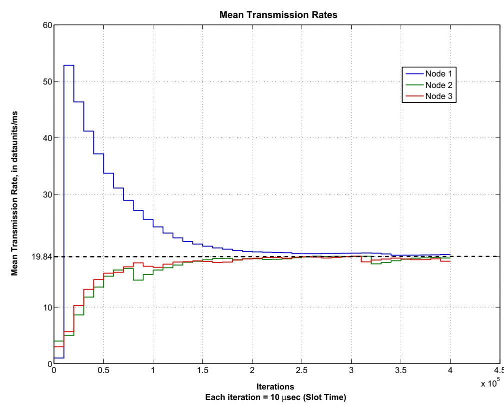

Fig. 3. Mean transmission rates of nodes 1, 2, and 3 transmitting to the same node 4. All nodes are in the sensing region. The online algorithm of (11) is used with $T = 100$ ms, $K = 5$, and $\delta T_s = 0.001$ ms.

The optimal value for the mean transmission rates for a 3-link network is calculated offline for the unconstrained problem of (8) for comparison purposes. The MATLAB built-in function `fminunc` is used for this purpose, and the optimal value for the mean transmission rates were obtained as

$$R_{1_{\text{opt}}} = R_{2_{\text{opt}}} = R_{3_{\text{opt}}} = 19.84 \text{ dataunits/ms}. \quad (15)$$

Fig. 3 indicates that mean transmission rates obtained from the online distributed algorithm of (11) converge to the optimal values, defined in (15).

The online algorithm (11) does not take into account the rate constraint defined in (14). The constrained convex nonlinear programming problem defined in (8), (13), and (14) is solved by optimizing the mean transmission rates R_k, $k = 1, 2, ..., n$, of the transmitting nodes in the network of Fig. 1 by a suitable numerical optimization algorithm. A MATLAB built-in function `fmincon` is used to solve the optimization problem by configuring it to use the interior point algorithm (cf. [13], [14]).

Once the mean transmission rates are optimized, they are fixed in a CSMA platform (developed in MATLAB) that uses the carrier sense channel access protocol. The function `fmincon` solves the optimization problem only for a set of feasible arrival rates. A slot time of 10 μs is used,

TABLE I

OPTIMAL VALUES OF THE MEAN TRANSMISSION RATES FOR A 2-LINK COLLISION NETWORK FOR VARIOUS VALUES OF SENSING DELAYS. THE OPTIMUM VALUES OF THE MEAN TRANSMISSION RATES ARE THE SOLUTION TO THE CONSTRAINED PROBLEM DEFINED IN (8), (13), AND (14).

	Max. Feasible Arrival Rate		Opt. Mean TX rate	
Sensing Delay	λ_1	λ_2	R_1	R_2
0.001	0.4	0.6	4.15	6.22
0.01	0.39	0.42	6.05	6.49
0.1	0.2	0.39	1.33	2.35

TABLE II

OPTIMAL VALUES OF THE MEAN TRANSMISSION RATES FOR A 3-LINK COLLISION NETWORK FOR VARIOUS VALUES OF SENSING DELAYS. THE OPTIMUM VALUES OF THE MEAN TRANSMISSION RATES ARE THE SOLUTION TO THE CONSTRAINED PROBLEM DEFINED IN (8), (13), AND (14).

	Max. Feasible Arrival Rate			Opt. Mean TX rate		
Sensing Delay	λ_1	λ_2	λ_3	R_1	R_2	R_3
0.001	0.22	0.31	0.3	7.31	10.28	9.95
0.01	0.13	0.21	0.24	6.54	10.19	11.49
0.1	0.12	0.12	0.1	2.26	2.26	1.94

and the mean transmission lengths of the packets, $1/\mu_k$, $k = 1,2,\ldots,n$, are set to 1 ms. Further, a stable (and feasible) set of arrival rates, in the sense that the queue lengths at the transmitting nodes are stable, are chosen before simulation.

A 2-link collision network is simulated using the platform explained above. The optimal values of the mean transmission rates, R_1 and R_2, are obtained and tabulated as shown in Table I for different values of the sensing delay δT_s. Note that the capacity of the channel is normalized to 1 dataunit/ms. The mean transmission lengths of the packets, $1/\mu_1 = 1/\mu_2 = 1$ ms.

A CSMA system with collisions is implemented in MATLAB. Fig. 4 shows the evolution of the queue lengths of the nodes 1 and 2 (refer to Fig. 1) for a sensing delay of $\delta T_s = 0.01$ ms. The optimal mean transmission rates ($R_1 = 6.05$ dataunits/ms, $R_2 = 6.49$ dataunits/ms) are generated by `fmincon`, and the stable arrival rates of $\lambda_1 = 0.16$ dataunits/ms and $\lambda_2 = 0.2$ dataunits/ms are used.

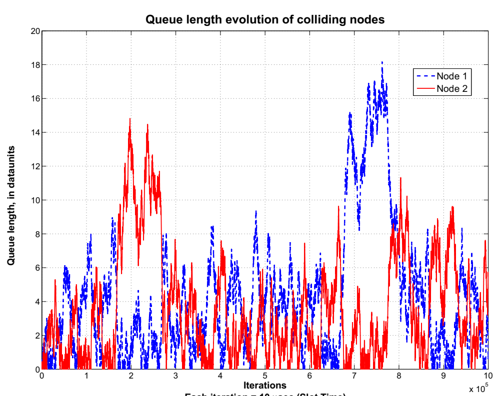

Fig. 4. Queue lengths of nodes 1 and 2 transmitting to the same node 3. The optimum values of the mean transmission rates are the solution to the constrained problem defined in (8), (13), and (14). All nodes are in the sensing region. $\delta T_s = 0.01$ ms, $R_1 = 6.05$ dataunits/ms, $R_2 = 6.49$ dataunits/ms, $\lambda_1 = 0.16$ dataunits/ms, $\lambda_2 = 0.2$ dataunits/ms.

A 3-link collision network is simulated similarly, and the optimal values of the mean transmission rates, R_1,R_2 and R_3, are obtained and tabulated as shown in Table II for different values of the sensing delay δT_s. Fig. 5 shows the evolution of queue lengths of the nodes 1, 2, and 3 (refer to Fig. 1) for a sensing delay of $\delta T_s = 0.01$ ms. The mean transmission lengths of the packets, $1/\mu_1 = 1/\mu_2 = 1/\mu_3 = 1$ ms. The optimal mean transmission rates ($R_1 = 6.54$ dataunits/ms, $R_2 = 10.19$ dataunits/ms, $R_3 = 11.49$ dataunits/ms) are generated by `fmincon`, and the stable arrival rates of $\lambda_1 = 0.01$ dataunits/ms, $\lambda_2 = 0.05$ dataunits/ms, $\lambda_3 = 0.02$ dataunits/ms are used.

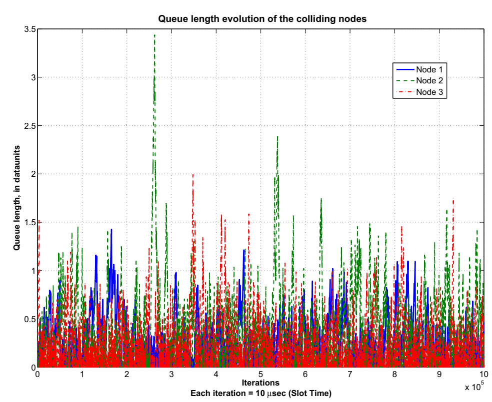

Fig. 5. Queue lengths of nodes 1, 2, and 3 transmitting to the same node 4. The optimum values of the mean transmission rates are the solution to the constrained problem defined in (8), (13), and (14). All nodes are in the sensing region. $\delta T_s = 0.01$ ms, $R_1 = 6.54$ dataunits/ms, $R_2 = 10.19$ dataunits/ms, $R_3 = 11.49$ dataunits/ms, $\lambda_1 = 0.02$ dataunits/ms, $\lambda_2 = 0.05$ dataunits/ms, $\lambda_3 = 0.05$ dataunits/ms.

The simulations are repeated 10 times for each of 2-link and 3-link collision networks, and the average (arithmetic mean) of the number of collisions is calculated for each case. Table III shows the average number of collisions when a set of optimized value of the mean transmission rates are used. The packet collisions in the network are reduced to less than 0.2% for the sensing delays listed in the table.

TABLE III

AVERAGE NUMBER OF COLLISIONS FOR A 2-LINK AND 3-LINK COLLISION NETWORKS FOR VARIOUS VALUES OF SENSING DELAYS. THE OPTIMUM VALUES OF THE MEAN TRANSMISSION RATES ARE THE SOLUTION TO THE CONSTRAINED PROBLEM DEFINED IN (8), (13), AND (14).

	Av. collisions using optimized Mean TX rates, in %	
Sensing Delay, δT_s, in ms	2-link	3-link
0.001	0.06	0.16
0.01	0.08	0.17
0.1	0.02	0.04

VI. CONCLUSION

A model for collisions is developed and incorporated in the continuous CSMA Markov chain. An online distributed algorithm for maximizing the collision-free transmission states is developed that estimates the rates from the steady-state distribution of the Markov states. To account for the rate constraints, a constrained optimization problem is defined, and a numerical solution is suggested. Simulation results infer that the average number of collisions by using the optimized parameters is reduced to less than 0.2%. Future efforts will focus on including queue length constraints in the optimization problem, and developing online solutions to the combined collision minimization and throughput maximization problem.

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