Efficient Spectrum Allocation in Multiband CSMA Networks*

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Abstract—We consider the problem of assigning a group of users with different rate requirements to a set of frequency bands, which may have different bandwidths, when the users access the channel via carrier-sense multiple access (CSMA). For example, in systems that employ dynamic spectrum access (DSA), secondary users may interleave their transmissions into space-time-frequency slots left open by primary users. The set of primary users that are active at a given time may leave a set of available channels that have unequal bandwidths. The use of CSMA by the secondary users reduces the need to coordinate transmissions among the users allocated to a particular frequency band, but it also results in potential collisions, which reduce the overall data rate that can be accommodated in the band. This is especially true when the presence of a finite sensing delay is considered. In this paper, we consider the layer problem of allocating users to the available frequency bands to minimize the bandwidth used (to accommodate other groups of secondary users), while taking into account the CSMA interactions of assigning multiple users to a band. We formulate this as a new form of bin packing problem, in which the size of the bin depends on the number of users that are assigned to the bin. A near optimal solution to this problem is found numerically using the Gurobi solver, and the performance is compared with the suboptimal first-fit algorithm, which has complexity that is appropriate for online implementation. Simulation results are provided to compare the optimal and online algorithms in terms of efficiency in allocating the bandwidth to the users and their complexity.

I. Introduction

We consider the problem of allocating users with different rate requirements to a set of available channels, such that the overall bandwidth used is minimized to allow for other users. This may be particularly useful in cognitive radio systems that employ dynamic spectrum access (DSA). For instance, in the DSA interleaving approach, secondary users are allocated to frequency bands that are not currently being used by some set of primary users. In this scenario, minimizing the bandwidth used by one set of secondary users will maximize the channel

bandwidth available to other secondary users. However, this will also often require allocating multiple users to a single frequency band, in which case the data rates that can be achieved will depend on the transmission rates of the users and on the rate at which collisions occur because of the finite delay in sensing when another user begins to transmit.

We formulate the problem of allocating a set of users with specified transmission rate requirements to a set of channels with specified channel bandwidths as a form of variable-sized bin packing problem (VBPP) [1]–[6]. Our formulation differs from previously considered VBPPs in that the number of bins is fixed, and because of the reduction in useable capacity caused by channel collisions in the CSMA MAC, the useable capacity of the bin depends on the number of users assigned to the bin and the rates of those users.

To determine whether a set of rates is achievable in a given bandwidth, we utilize the continuous-time Markov model for CSMA with finite sensing delays that was developed in [7]. However, an analytical expression for the region of achievable arrival rates is not possible because it is the solution to a nonlinear constrained programming problem. We employ the interior point algorithm (see [8]–[11]) to solve the optimization problem, and further provide an analytic expression that can be used to approximate the achievable arrival rate region when there are 100 or fewer users assigned to a frequency band. The analytical expression also has the advantage that it depends only on the number of users assigned to the channel and not on the rates of those users.

The bin-packing problem can be solved for a (near) optimal solution using the Gurobi solver but with relatively high complexity. Thus, we also use an online algorithm called the First Fit (FF) algorithm. We present results comparing the performance of the optimal and FF algorithms in terms of efficiency of spectrum use and complexity.

The rest of the paper is organized as follows: Section II describes the network model and the notations used throughout the paper. Section III explains the CSMA model with propagation delays, and introduces the throughput maximization problem. The formulation of the cross-layer problem for efficient channel allocation is described in Section IV. An online algorithm is described in Section IV-B for the problem, and simulation results are given in Section V to compare the online with the optimal algorithm.

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II. System Model and Problem Formulation

Consider a system of n mobile users that communicate to an Access Point (AP) over a wireless network as shown in Fig. 1.
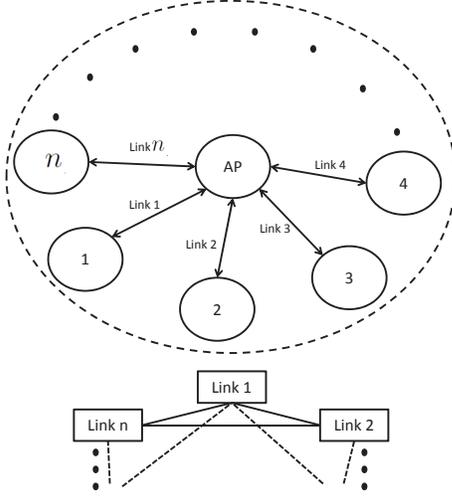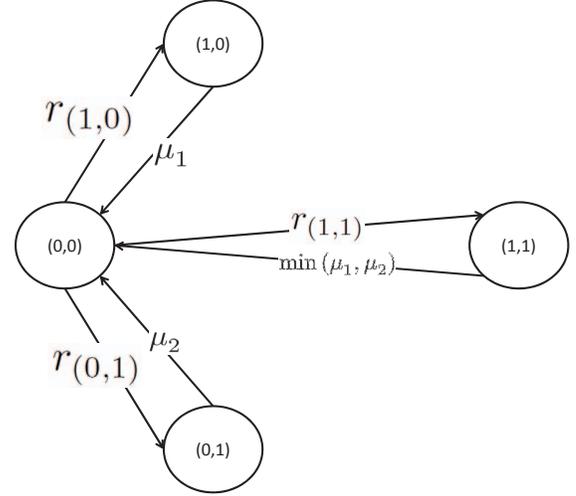
Fig. 1. A n-link network scenario and conflict graph.

Fig. 2. CSMA Markov chain for a 2-link network scenario.

The mobile nodes are allocated to a set of disjoint frequency bands. Multiple mobile node may be allocated to the same band, in which case the nodes use the CSMA protocol for multiple access. We denote the list of n user nodes (or pieces) as $L = \{1, 2, \ldots, n\}$. Let $\Lambda = \{\lambda_1, \lambda_2, \ldots, \lambda_n\}$, where λ_i is the transmission rate required by user i.

We formulate the problem of allocating the mobile users to a set of a available channels as a VBPP. Using the terminology of bin-packing problems, the items to be packed are the users, and the sizes of the items, $s(i)$ are the required rates $s(i) = \lambda_i$, $1 \leq i \leq n$. Without loss of generality, denote the available frequency bands (or subchannels) as $\mathbb{B} = \{1, 2, \ldots, m\}$, and let the maximum transmission capacity of channel i be C_i. However, because of collisions in CSMA, the maximum transmission capacity of a channel can be achieved only when a single user is assigned to the channel. A unique feature of the VBPP we consider is that the useable capacity of a given channel will depend on the number of users assigned to that channel and the rates of those users. In Section III, we provide an approximation for the useable capacity of a channel, and the approximation only depends on the number of users assigned to that channel. Let Λ_j denote the set of rates of the users assigned to channel j. Then $S(j; \Lambda_j)$ denotes the useable capacity of channel j when users with rates Λ_j are assigned to it.

Then the spectrum allocation problem can be formulated as the following VBPP:

$$\min_{J \subseteq \mathbb{B}, \{\Lambda_j\}} \sum_{j \in J} S(j, \Lambda_j) \qquad (1)$$

subject to

$$\bigcup_{j \in J} \Lambda_j = \Lambda \text{ and } \Lambda_i \cap \Lambda_j = \emptyset \ \ \forall i \neq j \qquad (2)$$

$$\sum_{\lambda \in \Lambda_j} \lambda \leq S(j, \Lambda_j), \quad j \in J. \qquad (3)$$

Constraint (2) requires that $\{\Lambda_j\}, j \in J$ form a partition of Λ, and (3) requires that the set of rates assigned to each channel is achievable with CSMA for the specified user allocation, as described in Section III.

III. CSMA CHANNEL CAPACITY WITH CARRIER PROPAGATION DELAYS

Users assigned to the same frequency band access the channel using CSMA. We assume that all of the nodes in the network can sense the transmissions of all of the other nodes, provided that two or more transmissions do not begin within a fixed propagation delay, δ. If two or more nodes initiate transmission within δ, there will be a collision, and all of the packets involved in the transmission are assumed to be lost. The backoff time of link k is modeled using an exponential distribution with mean $1/R_k$. The objective in this paper is to determine whether for a specified set of desired throughput rates Λ_j, there exist channel access rates R_k, $k = 1, 2, ..., n$, such that the throughput rates can be achieved.

The continuous-time Markov model for CSMA with fixed propagation delay δ from [7] is used to determine the achievability of a set of rates Λ with $|\Lambda| = n$. The state of the Markov model is denoted by $x^i \in \{0, 1\}^n$, where $x_k^i = 1$ if link k in state i is active, and $x_k^i = 0$ if link k in state i is idle. Two sets of indices for the collision-free transmission states, $\mathcal{A}$, and the collision states, $\mathcal{C}$, are defined below:

$$\mathcal{A} = \left\{ i \mid \sum_{k=1}^{n} x_k^i = 1 \right\}$$

$$\mathcal{C} = \left\{ i \mid \sum_{k=1}^{n} x_k^i > 1 \right\}$$

The Markov chain for a 2-link network scenario is shown in Fig. 2.

The stationary distribution of the continuous time Markov chain [7] is defined as

$$p_i\left(\mathbf{R}\right) \triangleq \frac{r_i}{\sum\limits_j r_j}, \tag{4}$$

where $\mathbf{R} \triangleq [R_1, R_2, ..., R_n]$ is the transmission rate vector, and

$$r_i \triangleq \begin{cases} \left\{ \dfrac{\sum\limits_{k=1}^{n}\left(x_k^i R_k \prod\limits_{l\neq k}\left(1-p_{c_l}\right)^{\left(1-x_l^i\right)}\right)}{\sum\limits_{k=1}^{n} x_k^i \mu_k} \right\} & \text{for } i \in \mathcal{A}, \\[2em] \dfrac{\sum\limits_{k=1}^{n}\left(x_k^i R_k \prod\limits_{l\neq k}\left(p_{c_l}\right)^{x_l^i}\left(1-p_{c_l}\right)^{\left(1-x_l^i\right)}\right)}{\min\limits_{m:x_m^i\neq 0}\left(\mu_m\right)} & \text{for } i \in \mathcal{C}, \\[2em] 1 & \text{otherwise,} \end{cases} \tag{5}$$

and $1/\mu_i$ is the mean transmission length of the packets if the network is in one of the states in set $\mathcal{A}$. The quantity p_{c_l} in (5) is the probability that link l becomes active within a time duration of δ from the instant link $k \neq l$ becomes active (see [7]). We assume that $\mu_i = 1 \ \forall i$.

To quantify the throughput, a log-likelihood function is defined [7] as the summation over all the collision-free transmission states as

$$F\left(\mathbf{R}\right) \triangleq \sum_{i\in\mathcal{A}} \ln\left(p_i\left(\mathbf{R}\right)\right) \tag{6}$$

$$= \sum_{k=1}^{n} \ln R_k - (n-1)\sum_{k=1}^{n} R_k\delta$$

$$- n \ln\left[1 + \sum_{k=1}^{n} R_k \prod_{l\neq k}\exp\left(-R_l\delta\right)\right.$$

$$\left. + \sum_{i\in\mathcal{C}}\sum_{k=1}^{n} x_k^i R_k \prod_{l\neq k}\left(p_{c_l}\right)^{x_l^i}\left(1-p_{c_l}\right)^{\left(1-x_l^i\right)}\right], \tag{7}$$

which can be shown to be concave in $\mathbf{R}$ [7]. Hence, the optimization problem[1] is defined as

$$\max_{\mathbf{R}}\left(F\left(\mathbf{R}\right)\right). \tag{8}$$

In addition to maximizing the log-likelihood function, certain constraints must be satisfied. Before proceeding with formulating the constraints, let $\Psi_k\left(\mathbf{R}\right)$ denote the service rate for a link k, defined as

$$\Psi_k(\mathbf{R}) \triangleq \frac{R_k \prod\limits_{l\neq k}\exp\left(-R_l\delta\right)}{\sum\limits_j r_j}, \tag{9}$$

$k = 1, 2, ..., n-1$, where r_j is defined in (4) and (5). Hence, the service rate at each transmitter of a link needs to be greater

[1]An alternate way to express the optimization problem is to define the log-likelihood expression in (6) as a function of the collision states, and minimize this function. It is worth noting that in such a case, we obtain a nonlinear concave minimization problem, which is tedious to solve.

than or equal to the arrival rate λ_k, i.e.,

$$\ln \Psi_k\left(\mathbf{R}\right) - \ln \lambda_k \geq 0, \tag{10}$$

$k = 1, 2, ..., n-1$, and the chosen mean transmission rates $R_k, k = 1, 2, ..., n-1$ need to be non-negative, i.e.,

$$R_k \geq 0, \tag{11}$$

Note that $\ln S_k\left(\mathbf{R}\right) - \ln \lambda_k = 0, \lambda_k > 0$ is concave for all k.

IV. EFFICIENT SPECTRUM ALLOCATION

The preceding section briefly discussed a model that was introduced in [7] for a dynamic delay-prone CSMA network, and calculated the optimized mean transmission rates for maximal throughput, expressed in terms of the steady-state probabilities of the collision-free transmission probabilities. As explained in Section I, CSMA cognitive radio technologies uses the spectrum sensing functionality to determine the interference of the primary users using the channel, and makes a decision if the secondary users can be accommodated in that channel. For this purpose, it is critical to analyze the achievable arrival rate region based on the CSMA policy (any policy that achieves the optimal mean transmission rates) used, and hence evaluate the DSA protocols based on such MAC characteristics. The following section explains the determination of this region.

A. Achievable Arrival Rate Region with CSMA Propagation Delays

The optimization problem defined in (8)-(11) is a concave constrained nonlinear programming problem, and obtaining a analytical solution, and further determining the achievable arrival rate region, is tedious. The Karush-Kuhn-Tucker conditions for the problem in (8)-(11) even for a simple 2-link case of the network gives a total of 6 nonlinear simultaneous equations, and obtaining an analytical solution for such a problem is difficult. There are numerical techniques adopted in the literature which have investigated nonlinear programming problems in detail [8]–[11]. We will use the interior point algorithm to solve the optimization problem defined in (8)-(11) for deriving the achievable arrival rate region. Hence, the achievable arrival rate region is obtained by varying the arrival rates until a maximum arrival rate vector is determined that numerically solves (i.e., determines the optimal set of transmission rates) the nonlinear convex programming problem defined in (8)-(11) using the interior point algorithm. From such numerical simulations using Matlab, the bin size $S(j, \Lambda_j)$ in (3) can be approximated based on the number of users assigned to the band as $S(j, \Lambda_j) \approx C_j \overline{S}(n)$. Here, $\overline{S}(n)$ is an approximation of the normalized useable capacity, and is the piecewise linear continuous function

$$\overline{S}(n) = \begin{cases} 1.11 - 0.11n, & 1 \leq n < 4, \\ 0.91 - 0.06n, & 4 \leq n < 7, \\ 0.7156 - 0.0322n, & 7 \leq n < 16, \\ 0.3714 - 0.0107n, & 16 \leq n < 30, \\ 0.0714 - 0.0007n, & 30 \leq n \leq 100, \\ 0, & n > 100. \end{cases} \tag{12}$$

It is worth noting that such expressions for the achievable arrival rate region can be obtained for a variety of MAC systems such as IEEE 802.11 (see [12] for experimental evaluation of this region). The development in the current and the previous sections provides a recipe for efficient DSA, and can be applied to a wide range of MAC systems if the achievable arrival rate region can be approximated as in (12).

Substituting for $\overline{S}(n)$ using (12), the quantity $S(j; \Lambda_j)$ in (1) can be expressed using the Special Ordered Set of type 2 (SOS-2) representation for piecewise continuous linear function, given in the right-hand-side of (12). Note that this problem will be a variant of the classical bin packing problem [1], [13]–[16], also a Mixed Integer Quadratic Programming (MIQP) problem in this case. The bin packing problem is NP-Hard [13]. We use the Gurobi solver [17] for determining the optimal channel allocations, and will refer to the procedure used by it as OPT. The results for various instances of this problem will be discussed subsequently in Section V.

The solutions provided by the solver is a centralized, offline solution to the channel allocation problem. Further, the time complexity for such problems are very high as will be seen in Section V. In order for such algorithms to find applications that involve dynamic allocations of radio nodes to a spectrum, such as in the case of cognitive radio technologies, it is essential to adopt online approaches to solve the MIQP. We will discuss the available online algorithms in the following section.

B. Online Algorithms

Spectrum sensing in cognitive radio technologies (for details, see [18], [19]) allows the arriving user node to detect and choose the RF spectrum based on the underlying MAC model described in Section III. Hence, as noted earlier, online algorithms are required for such capabilities. Note that we intend to solve a variant of the bin packing problem, called the Variable-sized Bin Packing Problem (VBPP) for obvious reasons of having to deal with finite number of distinct bandwidths. Various schemes have been proposed for offline and online algorithms [1]–[6] for VBPP.

A commonly used online algorithm that can be used to solve this VBPP is the First Fit (FF) procedure (cf. [2], [3], [20]) algorithm, that reviews all partially packed bins, placing the next available piece in the lowest indexed bin if there is room, before starting to pack a new bin. The time complexity for FF is $\mathcal{O}(n \log m)$ [2]. Other commonly used algorithms to solve VBPP includes Variable Harmonic (procedure VH) algorithm [4] with a time complexity of $\mathcal{O}(n \log m)$. We will use FF to solve our problem, as the time-complexity is comparable with that of VH with fixed number of available bins. It is important to note that certain instances might result in infeasibility with online algorithms, especially with a finite set of bins. The analysis of the online algorithms in the context of determining the asymptotic and absolute worst-case performance ratios with respect to the optimal algorithm (cf. [14]) for this particular problem is left for future work.

To implement FF, a binary tree is generated with $\log m$ levels and m bins, each of the nodes in the tree has a label denoting the size of the bin [2]. The tree has a total of $2m-1$ nodes - m leaves corresponding to the m bins, and $m-1$ inner nodes, as shown in Fig. 3. The labels of the leaf nodes of the initial tree denote the capacity of its corresponding bin (bins are sorted in descending order from the leftmost to the rightmost leaf) for the first node to be packed. The labels of each of the inner nodes denote the maximum value of its child nodes. Once the tree is obtained, procedure FF (see Algorithm 1) is used for allocating the pieces to the bins. Each of the piece will start at the root node, and will always start comparing its value with the label of the leftmost child node. If the piece size is lesser than the label, the piece traverses; otherwise, checks the label of the right child node and traverses accordingly. Similar procedure is employed for the subsequent levels, and once the piece reaches the leaf, it is packed in the corresponding bin. The number of comparisons for this effort is $\log m$. Once packed, the corresponding leaf node is updated with the remainder capacity for the *next* piece, calculated by using the approximation, $S(j, \Lambda_j) \approx C_j \overline{S}(n)$, in (12), and the cumulative arrival rates of the pieces that has already been accommodated in that bin. Next, the inner nodes of the tree are updated by selecting the maximum value of its child nodes. The label updating process takes $\log m$ steps. Hence the time complexity to pack n pieces is $\mathcal{O}(n \log m)$. A value of 0 for the leaf nodes of the tree indicates that the corresponding bin cannot accommodate another piece for it has used up its capacity for the packed pieces.

V. Simulation Results

DSA protocols can be characterized by various performance metrics such as time-complexity and bandwidth utilization. Online algorithms like FF rely on the order of arrivals of the users and their sizes and hence, the worst-case performance ratios with respect to the optimal algorithm will depend on them. Such analysis for FF for the classical bin packing problem is detailed in [14]. In our problem, it is difficult to establish such metrics due to the dependence on the numerical achievable arrival rate model of (12), in addition to the dependence on the order of user arrivals. Hence, for evaluating the performance of FF for DSA applications, we determine the average bin space used for a given set of bandwidths subsequently.

Numerical simulations are performed using Matlab and Gurobi solver in a Intel® Core™ i5 CPU 750, 64-bit 2.67 GHz processor to compare the performance of the online algorithm FF with the optimal algorithm OPT (Gurobi solver is used to determine the OPT algorithm). For this purpose, we fix the number of users n waiting to be allocated spectrum, and chose their arrival rates from a uniform distribution from the set $(0, 10]$ MHz. Then, the utilized bandwidth to allocate these users is calculated using OPT and FF. We repeat this procedure for a total of 100 simulations for each n, varied between 5 and 15, with the user nodes arriving based on a uniform distribution from the set $(0, 10]$ MHz in each simulation, and
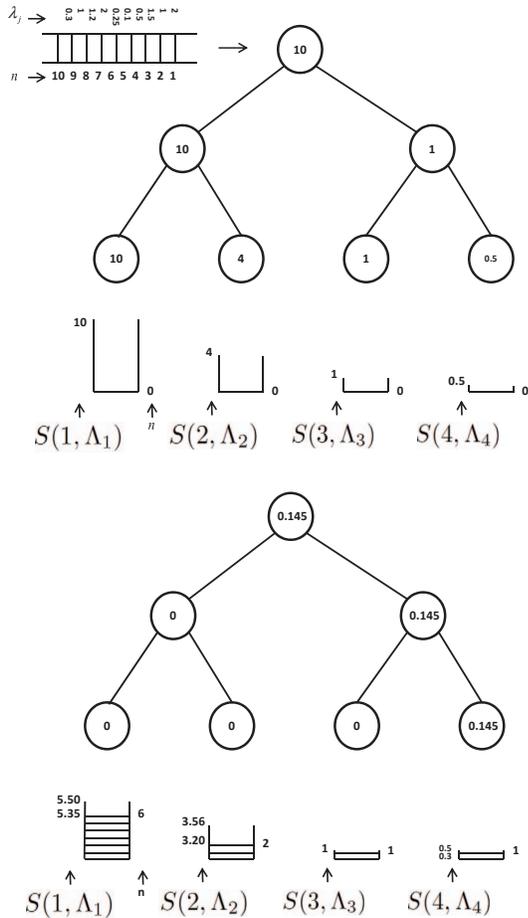
Fig. 3. Binary tree data structure for implementation of FF. Initial tree (top) with the nodes waiting to be packed. The size of the bins, i.e., $C = \{10, 4, 1, 0.5\}$ MHz, and the piece arrival rates, i.e., $s[i] = \{2, 1, 1.5, 0.5, 0.1, 0.25, 2, 1.2, 1, 0.3\}$ MHz. The capacity of the bins, i.e., $S(j; \Lambda_j)$, $1 \leq j \leq 4$, is approximated in Section IV-A. Final tree (bottom) with packed bins, after using FF in Algorithm 1. A value of 0 at the leaf nodes in the final tree implies that the bin can no longer accept nodes of *any* given size, given that a set of nodes with specified arrival rates has been packed in that bin, and a value of 0.145 implies that a node with $\lambda = 0.145$ can be accommodated in the bin C_4.

Algorithm 1: FF

Input: m; n; `BinaryTree`(m);
foreach $i \in$ `LeafNode` **do**
  $N(i) = 0$;   % All bins have 0 nodes initially.
  $L(i) = 0$;   % Size of occupance of nodes is 0 initially.
end
`Temp` = 0;
foreach $j \in \{1, 2, .., n\}$ **do**
  `Temp` = 0;
  $l \leftarrow$ `Root`;
  while $l \notin$ `LeafNode` **do**
    if $s[j] \leq s[l]$ **then**
      if $s[j] \leq s[$`LeftChild`$(l)]$ **then**
        $l =$ `TraverseLeft`(l);
      else
        $l =$ `TraverseRight`(l);
      end
    else
      The node cannot be packed;   % Infeasible solution.
      `Temp` = 1;
      break;
    end
  end
  if $l \in$ `LeafNode` & `Temp` = 0 **then**
    $N(l) = N(l) + 1$;
    $L(l) = L(l) + \lambda_j$;
    $c =$ `Capacity`$(N(l) + 1)$;   % Capacity for the next node.
    $s[l] = c - L(l)$;   % Remainder capacity.
    while $l \neq$ `Root` **do**
      if $s[l] < s[$`Adjacent`$(l)]$ **then**
        $s[$`Parent`$(l)] = s[$`Adjacent`$(l)]$;
      else
        $s[$`Parent`$(l)] = s[l]$
      end
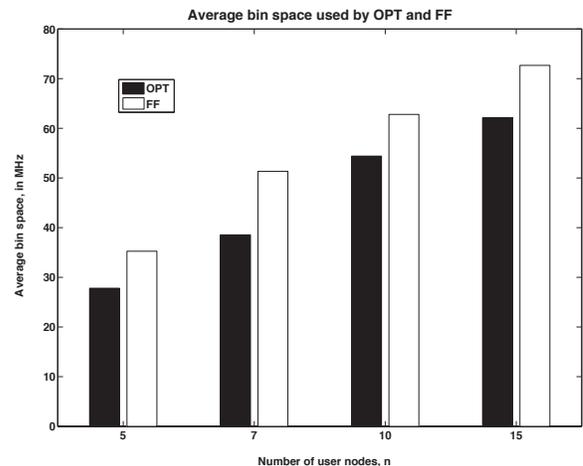 $l =$ `Parent`(l);
    end
  end
end

Fig. 4. Average bin space used by OPT and FF. The arrival rates are chosen uniformly from the set $(0, 10]$ MHz. The size of the bandwidths, $C = \{50, 25, 20, 10\}$ MHz.

the average value of the utilized bandwidth (or bin space) is determined using OPT and FF for each n. The comparison of the average bin space used by OPT and FF is shown in Fig. 4.

Further, to establish the time complexities of OPT and FF, we start with a set of users waiting to be assigned a bandwidth in one of the four spectrums $\{50, 25, 20, 10\}$ MHz. The arrival rates for these users were selected from a uniform distribution from the set $(0, 3]$ MHz. The elapsed time for the two algorithms was determined using Matlab. The process is repeated for the number of users varied between 1 and 100. Fig. 5 shows the time complexities of the two algorithms. As expected, solving the MIQP defined in has a higher time complexity, especially when $n > 15$ (close to 11 minutes to determine the optimal solution for $n = 20$).

One of the drawbacks of using an online algorithm if one does not analyze the algorithm in the context of determining the asymptotic and absolute worst-case performance ratios with respect to the optimal algorithm, as mentioned in Section IV-B, is that there might exist instances of infeasibility for a certain set of users with certain arrival rates waiting to be allocated in one of the spectrums having a certain bandwidth.
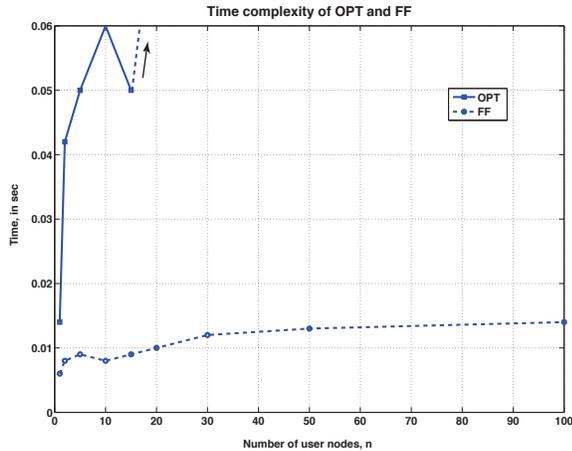
Fig. 5. Time complexity of OPT and FF. The arrival rates are chosen uniformly from the set $(0, 3]$ MHz. The size of the bandwidths, $C = \{50, 25, 20, 10\}$ MHz.

TABLE I

TABLE SHOWING THE INFEASIBILITY OF AN INSTANCE USING THE ONLINE ALGORITHM FF WHEN OPT IS FEASIBLE. THE NUMBER OF BINS, $m = 4$, NUMBER OF PIECES, $n = 10$, AND $C = \{10, 4, 1, 0.5\}$ MHZ. SEE FIG. 3 FOR ILLUSTRATION.

Arrival Rates, in MHz	Bin space used, in MHz		Comments
	OPT	FF	
$\{2, 1, 1.5, 0.5, 0.1, 0.25, 2, 1.2, 1, 0.3\}$	10.11	10.56	Feasible by OPT and FF
$\{2, 1, 1.5, 0.5, 0.1, 0.25, 2, 1.2, 1, 1.3\}$	10.88	10.06	p_{10} is infeasible using FF

Table I shows one instance of such infeasibilities.

VI. CONCLUSION

We developed techniques to optimize the allocation of users with specified rates to multiple channels of specified bandwidths, where users that are assigned to the same channel use CSMA. We formulate this optimization problem as a variable-size bin packing problem with some novel twists, including that the bin size is dependent on the items being packed into the bin. The bin size corresponds to the achievable rate region for the given bandwidth, and we determine the optimal rate region using a continuous-time CSMA MAC model with propagation delays. The interior point algorithm is used to determine the optimal mean transmission rates of the user nodes, and we further approximate the achievable arrival rate region of the network as a piecewise continuous linear-in-n function, which is used in the bin-packing optimization. The Gurobi solver is used to provide a (near) optimal solution to the bin-packing problem, and the first-fit algorithm is used to find an online, approximate solution. suggested for solving the MIQP. simulations are used to compare the performances of the algorithms in terms of efficiency of bandwidth utilization and complexity.

REFERENCES

[1] D. Friesen and M. Langston, "Variable sized bin packing," *SIAM J. Computing*, vol. 15, no. 1, pp. 222–230, 1986.
[2] D. Johnson, *Near-optimal Bin Packing Algorithms*, ser. Massachusetts Institute of Technology, project MAC. Massachusetts Institute of Technology, 1973.
[3] ——, "Fast algorithms for bin packing," *J. Computer Sys. Sciences*, vol. 8, no. 3, pp. 272–314, 1974.
[4] J. Csirik, "An on-line algorithm for variable-sized bin packing," *Acta Informatica*, vol. 26, no. 8, pp. 697–709, 1989.
[5] C. C. Lee and D. T. Lee, "A simple on-line bin-packing algorithm," *J. ACM*, vol. 32, no. 3, pp. 562–572, Jul. 1985.
[6] S. S. Seiden, "On the online bin packing problem," *J. ACM*, vol. 49, no. 5, pp. 640–671, Sep. 2002.
[7] S. Subramanian, E. L. Pasiliao, J. M. Shea, M. M. Carvalho, and W. E. Dixon, "Throughput maximization in CSMA networks with collisions," in *Proc. IEEE Mil. Commun. Conf.*, Orlando, FL, 2012, pp. 908–913.
[8] D. P. Bertsekas, *Nonlinear Programming*. Belmont, MA: Athena Scientific, 1999.
[9] D. G. Luenberger, *Introduction to Linear and Nonlinear Programming*. Addison-Wesley, 1973.
[10] M. S. Bazaraa, H. D. Sherali, and C. M. Shetty, *Nonlinear programming - Theory and algorithms (2nd Edition)*. Wiley, 1993.
[11] S. Boyd and L. Vandenberghe, *Convex Optimization*. New York, NY, USA: Cambridge University Press, 2004.
[12] A. Jindal and K. Psounis, "The achievable rate region of 802.11-scheduled multihop networks," *IEEE/ACM Trans. Netw.*, vol. 17, no. 4, pp. 1118–1131, Aug. 2009.
[13] S. Martello and P. Toth, *Knapsack problems: Algorithms and computer implementations*. New York, NY, USA: John Wiley & Sons, Inc., 1990.
[14] E. Coffman, Jr., M. Garey, and D. Johnson, "Approximation algorithms for NP-hard problems," D. Hochbaum, Ed. Boston, MA, USA: PWS Publishing Co., 1997, ch. Approximation algorithms for bin packing: A survey, pp. 46–93.
[15] A.-C. Yao, "New algorithms for bin packing," *J. ACM*, vol. 27, no. 2, pp. 207–227, Apr. 1980.
[16] S. Seiden, "An optimal online algorithm for bounded space variable-sized bin packing," *SIAM J. Discrete Math.*, vol. 14, p. 2001, 2000.
[17] I. Gurobi Optimization, "Gurobi optimizer reference manual," 2012.
[18] I. Akyildiz, W.-Y. Lee, M. Vuran, and C. Mohanty, "NeXt generation/dynamic spectrum access/cognitive radio wireless networks: A survey," *Comp. Net.*, vol. 50, no. 13, pp. 2127–2159, 2006.
[19] ——, "A survey on spectrum management in cognitive radio networks," *IEEE Commun. Mag.*, vol. 46, no. 4, pp. 40–48, 2008.
[20] N. Kinnerseley and M. Langston, "Online variable-sized bin packing," *Discrete Applied Mathematics*, vol. 22, no. 2, pp. 143–148, 1988-89.