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ARTICLES

A Naturalistic Exploration of Forms and Functions of Analogizing

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The purpose of this article is to invigorate debate concerning the nature of analogy, and to broaden the scope of current conceptions of analogy. We argue that analogizing is not a single or even a fundamental cognitive process. The argument relies on an analysis of the history of the concept of analogy, case studies on the use of analogy in scientific problem solving, cognitive research on analogy comprehension and problem solving, and a survey of computational mechanisms of analogy comprehension. Analogizing is regarded as a macrocognitive phenomenon having a number of supporting processes. These include the apperception of resemblances and distinctions, metaphor, and the balancing of semantic flexibility and inference constraint. Psychological theories and computational models have generally relied on (a) a sparse set of ontological concepts (a property called "similarity" and a structuralist categorization of types of semantic relations), (b) a single form category (i.e., the classic four-term analogy), and (c) a single set of morphological distinctions (e.g., verbal vs. pictorial analogies). This article presents a classification based on a "naturalistic" exploration of the variety of uses of analogical reasoning in pragmatically distinct contexts. The resultant taxonomy distinguishes pre-hoc, ad-hoc, post-hoc, pro-hoc, contra-hoc, and trans-hoc analogy. Each will require its own macrocognitive modeling, and each presents an opportunity for research on phenomena of reasoning that have been neglected.

Residing as it does "at the core of cognition" (Hofstadter, 2001, p. 499) analogy has been an important topic in the history, philosophy, and psychology of science, in cognitive science, and in the field of artificial intelligence (see Barnden & Lee, 2001; Gentner, Holyoak, & Kokinov, 2001; Komendzinski, 2001). The purpose of this article is to stimulate debate concerning the nature of analogy, to present some cautionary tales, and to strengthen research by broadening its

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scope based upon a naturalistic inquiry. First, we briefly recapitulate the history of the concept of analogy and the received views on the importance of analogy. The second section expresses one important culmination of history—the tendency to reify analogy as a single form and as a single, fundamental mental process. This leads in the third section to a naturalistic or functional classification, pointing toward new horizons for research and modeling.

A BRIEF HISTORY OF THE CONCEPT OF ANALOGY

The word *analogy* comes from the Greek term for geometric or numerical proportions, ratios, or symmetries. That is, it referred to the arrangement of two sets of numbers or geometrical forms such that the numbers or forms within each set are related by the same mathematical operator, transformation, or scale (*ana logos* = "same logic" or "according to a ratio"). Examples would be the numerical analogy, 1 : 2 :: 2 : 4, and the geometrical analogy expressed verbally as Circle : Sphere :: Triangle : Cone. Aristotle, among others, extended the analogy concept to denote the general process of selecting or perceiving resemblances or similarities, especially similarity of function (Ross, 1910, 1928).

Throughout the Middle Ages, rhetoricians generally regarded analogy as being on par with syllogism, as one of the major forms of proper argumentation. The concept of analogy was then broadened further—applied by grammarians as an explanation of historical change of word forms and inflections (similar meanings should be represented by similar forms). Analogy as a basic process in language change "was for a thousand years the preoccupation of the clearest heads in Greece and Rome" (Esper, 1973, p. 2).

As an analytical tool, as opposed to a purely rhetorical device, the analogy concept was expanded greatly over the centuries. In his *System of Logic* (1878, chap. 20), John Stuart Mill echoed Etienne de Condillac's definition of analogy, "the resemblance of relations." This definition is essentially the one that has been employed by modern philosophers, computer scientists, and cognitive scientists.

A broadened concept of analogy was utilized explicitly and productively throughout the Renaissance, as, for example, in Galileo's (1630/1953) explanation of his observations of the motions of the planets and their moons, Kepler's (early 1600s) astronomical and mathematical investigations, and Boyle's chemical researches (late 1600s). Analogy was a salient topic in Francis Bacon's (1620/1960) discussions of scientific inference. Analogizing has been an important topic in the history, philosophy, and psychology of science, in cognitive science, and in the field of artificial intelligence (AI; see Gentner, Holyoak, et al., 2001; Holyoak & Barnden, 1994a,b; Komendzinski, 2001).

Analogy, in one form or another, has widely been assumed to be a critical process in perception, language, learning, and problem solving (Carbonell, 1982; Chalmers, French, & Hofstadter, 1992; Holyoak, 1984; Hoyloak & Thagard, 1997; Mitchell, 1993; Petrie, 1979; Schumacher & Gentner, 1988; Verbrugge & McCarrell, 1977).

Going even further, it is often claimed that analogy is a fundamental, basic, or irreducible mental operation or process. Bushels of quotations show how widely held this view is. A sampler is presented in Table 1.

There is, to be sure, research showing that analogy is not ubiquitous and is not always helpful in problem solving, whether on the part of participants in laboratory research using comprehension tasks (Donnelly & McDaniel, 1993; Kolodner, 2002; Perkins, 1983), on the part of professionals who are engaged in brainstorming (Gordon, 1961, see p. 234), on the part of advanced students who are learning complex concepts (Spiro, Feltovich, Coulson, & Anderson, 1989), or

TABLE 1
Some Expressions of the Widely Held View that Analogy is a Fundamental Mental Operation

"Men reason by analogy long before they have learned to reason by abstract characters" (James, 1890, Vol. 2, p. 363).
"All expressions of mental phenomena are borrowed from analogous material in experience" (Höffding, 1904, p. 153).
"Analogy is a primordial condition of all language" (Breal, 1964, p. 7).
Analogy may be equated with stimulus generalization, isomorphism, metaphor, abstraction, transfer, and the scientific model" (Haskell, 1978, p. 161).
"Learning is a process of analogy-based transfer which necessitates the creation of new schemas and the 'tuning' of old ones" (Rumelhart & Norman, 1981, p. 357).
"Analogy is ubiquitous in human thinking" (Thagard et al., 1990, p. 259).
"A faculty for analogical reasoning is an innate part of human cognition. The concept of an inferentially sound analogy is universal" (Gentner & Jeziorski, 1993, p. 447).
"The ability to perceive similarities and analogies is one of the most fundamental aspects of human cognition" (Voisniadou & Ortony, 1989, p. 1).
"The basic constraints of analogy—similarity, structure and purpose—are already present in the chimpanzee. . . . preschool children, without any formal training, have a natural capacity to reason by analogy. . . . Analogical thinking is the product of evolutionary changes in the way animals represent knowledge of relations" (Holyoak & Thagard, 1995, pp. 4, 67, 72).

on the part of domain experts who are attempting to solve real-world problems (Bearman, Ball, & Ormerod, 2007; Klein & Calderwood, 1988).

Nevertheless, analogizing can be a useful reasoning strategy. But the literature goes beyond such a plain statement to the claim that analogy is a fundamental, basic, or irreducible mental operation or process (e.g., Rumelhart & Norman, 1981). The rationale for this big leap—regarding analogy as a fundamental cognitive operation—comes largely from the many studies of the uses of analogy in science.

ANALOGY IN SCIENCE

Creative analogy involves imagery, metaphor, the search for meaning, induction, and related macrocognitive phenomena (Hoffman, 1980, 1995; Hoffman, Cochran, & Nead, 1990). The importance of analogy to mature reasoning is underscored in countless retrospective analyses of creative scientific problem solving (Dunbar, 1996; Gentner et al., 1997; Millman & Smith, 1997). "The role of analogy in science can scarcely be overestimated" (Sternberg, 1977b, p. 99). Indeed, it is possible to analyze the broad history of all sciences in terms of the major analogies (Leatherdale, 1974, Thagard, 1993). Some examples of salient analogies and metaphors in the history of science are presented in Table 2.

As far as we can tell there exists no such thing as a modern philosopher or psychologist of science who has *not* argued or at least mentioned that analogies (or metaphors) are in some way essential to science.[1] Numerous studies have been conducted on how scientists create and use analogies in their

[1]For examples, see Berggren (1962); Broadbeck (1968); Campbell (1953); Carnap (1963); Chapanis (1961); Dreistadt (1968); Gentner and Grudin (1985); Gruber (1980); Harre (1970); Hoffman, Cochran, and Nead (1990); Hutten (1954); Knorr-Cetina (1981); Koestler (1964); MacCormac (1976); Nickles (1980); Oppenheimer (1956); Pap (1962); Ramsey (1964); Ricoeur (1975); Schön (1963); and Tweney, Doherty, and Mynatt (1982).

TABLE 2
Some Salient Analogies in the History of Science

Sound moves in waves, like water (Vitruvius).
A planet is a projectile (Newton).
Chemical elements can be arranged according to their properties like the suits in a deck of cards (i.e., the periodic table; Mendeleeff).
Gases are like a container of billiard balls (i.e., kinetic theory; Boyle).
Molecules can have shapes, like that of a snake (Kekule).
Division of the atomic nucleus is like cell fission (Frisch, Meitner).
The heart is a pump (Harvey).
The body/brain is a machine (Descartes, de Condillac).
Society is like an organism (le Bon).
Organisms are like a society (i.e., cell theory; Virchow).
The brain is like a telegraph/telephone switchboard (Helmholtz, Wundt).
The mind/brain is a control mechanism (de la Mettrie, Wiener).
The rational mind is a logical or symbol-processing machine (Pierce, Boole, Newell, Simon, McCarthy, Feigenbaum, Minsky, others).

scientific work, on how students use analogy in the learning of science (Chinn & Brewer, 1993; Donnelly & McDaniel, 2000; Kolodner, 2002; Newby, Ertmer, & Stepich, 1995) and how scientists' analogies are used in the teaching of science. This includes many studies of analogical transfer of abstract principles across domains, as influenced by literal or superficial similarity and the formation of misconceptions (e.g., Bassok, 1996; Goldstone & Sakamoto, 2003; Spiro et al., 1989).

ANALOGY AND COMPUTER SCIENCE

Motivated in part by the manifest importance of analogy, dozens of attempts have been made to computationally model analogical reasoning, spanning many applications—story understanding, theorem proving, means–ends analysis in planning, the resolving of political disputes, the solving of letter-string analogies, the solving of geometric analogies, problem solving in hydraulics, problem solving in thermodynamics, problem solving in process control, the use of analogy in automated deduction and machine learning, the use of analogy to extend program language concepts, and so on. (Because of the diversity of mechanisms and applications, a detailed analysis of AI models for analogy is well beyond the reach of this article. For reviews and more detailed treatments of AI models see Bergmann, 1979; Eliot, 1986; Hall, 1988; Hoffman 1995; Hofstadter, 1995; Holyoak & Barnden, 1994a; Indurkhya, 1992; Martin 1990; Prieditis, 1988; Russell, 1986, 1992; Stelzer, 1983; Steinhart, 1994; Way, 1991.)

Critical in most theories and models of analogy is a distinction between features and relations. It seems clear that both are involved in analogy, and that the sharing of features and relational meaning is critical to analogy creation and comprehension. When we say, for instance, that atoms are like solar systems, we do not mean that electrons share properties with planets, but that they share a relation, that is, they orbit nuclei just as planets orbit the sun. Anomalous analogies such as River : Story :: Fish : Milk effectively block the comprehender from determining any shared literal properties that might suggest a successful completion; relations are about all that is left. In experiments in which people are asked to solve anomalous analogies and then justify their solutions (M. G. Johnson, 1975; M. G. Johnson & Henley, 1992), relational solutions were the most frequent, and furthermore, independent judges show near-perfect agreement in rating the acceptability of the different solutions and in categorizing the solutions into relational subtypes. (This finding points to the problem of “semantic infinity” [Hoffman & Honeck, 1976] as a particular challenge for models of analogical reasoning. We will have more to say on this later in this article.)

Nearly all computational models rely on a process whereby hierarchies or graphs are matched on the basis of similarity (shared features and shared relations), empty feature slots are filled by a process of mapping or filtering, or there is some process of restructuring of relations (Burns, 1995; Forbus, Gentner, & Law, 1995; Hall, 1988; Holyoak, Novick, & Melz, 1994; Markman & Gentner, 2000). AI models include other techniques, such as constraint satisfaction (Thagard, Holyoak, Nelson, & Gochfeld, 1990). The many AI systems utilize a great variety of paradigms, including the deductive paradigm, the production rule paradigm, sequential versus parallel processing, Bayes nets, neural nets, and so on. Some systems are primarily based on syntax, some begin with elaborate memory representations, and yet others begin impoverished and then build up the representations. Some operate on individual analogies but others construct complex domain–domain mappings based on experience with multiple analogies. Some models seem passive; that is, they generate all possible mappings or pairings of elements and then evaluate them for coherence and pare down the set of good candidates. But others seem more active or selective, say, by restricting mappings to those that satisfy given a priori goals. Some systems painfully remember all of their past failures; other systems attempt to derive generalizable rules. Some check for the adequacy of a solution by soliciting information from the user; some do not. And so on.

Analogical reasoning has also been the subject of numerous laboratory investigations by psychologists.

PSYCHOLOGY OF ANALOGY

Psychometricians have used analogy problems in intelligence tests since the very first such tests were devised. Also in the study of individual differences, the four-term analogy completion task is typically used to gauge capacity for inductive reasoning (Pellegrino & Glaser, 1979). The ability to solve simple analogies correlates fairly well with scores on tests of general intelligence (Spearman, 1923; Sternberg, 1977b; Thurstone, 1938), the correlations ranging from about 0.45 to 0.82 across various studies.

Analogical reasoning has been the subject of numerous investigations using a variety of materials—cartoon depictions of hypothetical animals, patterns of colored squares, and of course the basic four-term verbal analogy, such as Robin : Bird :: Mustang : ? A variety of tasks have been used, including feature mapping, inference making, reaction time, recall, and recognition (e.g., Perrott, Gentner, & Bodenhausen, 2005). A main goal of this research has been to develop componential stage theories of analogy comprehension and transfer. There are bushels of findings.

Since four-term analogy problems are well-structured, they have been utilized in reaction-time and judgment tasks in an effort to disclose cognitive processes. Reaction times and error probabilities in the solving of simple geometric analogies are a function of the number of transformation operations involved (e.g., Mulholland, Pellegrino, & Glaser, 1980; Pellegrino &

Glaser, 1981). If the terms in simple verbal analogies are rearranged from the expected order, as in Deep : Costly :: Shallow : ?, then reaction time is inflated and error rates increase, reflecting the mental reordering of the terms (Sternberg, 1977b, chap. 5; see also Ace & Dawis, 1973, Barnes & Whitely, 1981).

Studies have been conducted on the conditions under which people will make analogical inferences during comprehension (e.g., Day & Gentner, 2007). In the simplest task, in which people are given analogies to solve, it is harder for people to complete three-term analogies than to find a correct completion in a given set of alternatives in a multiple-choice format (D. M. Johnson, 1962). People perform much better if they are given some explicit instruction, with examples of the analogy relation, and feedback on their responses (Inzer, 1972; Sternberg, 1977b, chap. 5). They do better on verbal analogies if the correct solutions are high-frequency associates of the given terms (Wilner, 1964).

Numerous studies have been conducted on the conditions under which people will solve problems by analogical transfer, for example, whether the noticing of analogies can occur in the laboratory or whether it depends on giving people hints, diagrams or other reasoning aids (e.g., Gick & Holyoak, 1980, 1983; Novick & Holyoak, 1991; Zamani & Richard, 2000). Transfer is most likely when surface or featural similarity is high and when there is only a brief delay between acquisition and test phases (cf. Wharton et al., 1994). Many studies have investigated the relative importance of relational versus featural similarity in the formation of analogical mappings (e.g., Gentner & Kurtz, 2006).

Lying close to the extreme of conceptual difficulty are the analogies of the Miller Analogies Test (MAT; Psychological Corporation, 1993).[2] The MAT is intended to assess scholastic aptitude at the graduate level; it emphasizes the recognition of verbal/conceptual semantic relations and fine shades of relational meaning. An example is

Annoy : Enrage :: Enlarge : (a. increase, b. exaggerate, c. augment, d. reduce).

If one were to assume that the first two terms express synonymy, the dilemma is that both a and c would work as answers. The “correct” solution is b, expressing the relation of “to do X but to a greater degree.” Scores on the MAT are correlated primarily with the vocabulary subtests in general intelligence tests (Guilford, 1967; Meer, Stein, & Geertsma, 1955), and are correlated more highly with tests of verbal analogy reasoning than with tests of pictorial or geometrical analogy reasoning (Sternberg, 1977a,b), suggesting that the MAT taps into both vocabulary and esoteric knowledge of semantic relations.

Much psychological research has involved preformulated problems that are well structured, semantically limited, and context free. This is the starting point for the Naturalist. This A : B :: C: D form is not the only form and function that analogy takes; different types of analogy occur in different contexts, and are used for entirely different purposes.

NATURAL CATEGORIES OF ANALOGICAL REASONING

Our search for a natural taxonomy began with a simple realization. Psychologists had been conducting studies in which participants are presented with preformulated analogy puzzles in

[2]“Miller Analogies Test” is a registered trademark of the Psychological Corporation, San Antonio, TX.

TABLE 3
Functionally Distinct Contexts for Analogy

Kind	*Context*	*Intended Use or Function*
Ad-hoc (*to this*) analogy	Creating analogies for use as analogies.	Analogies to suit particular purposes (e.g., intelligence tests, puzzles for use in laboratory experiments on problem solving, etc.)
Pre-hoc (*before this*) analogy	Creating analogies for use in discovery, explanation, or sensemaking.	Attempts to explain the unknown in terms of the known.
Post-hoc (*after this*) analogy	Analysis of pre-existing analogies, presented as incomplete analogies.	Comprehension of analogies by generating structural descriptions and cognitive models, and implementing computational approaches.
Pro-hoc (*about this*) analogy	Analytical reasoning about analogy as a concept.	Assessment and refinement of post-hoc models of reasoning.
Contra-hoc (*against this*) analogy	Creation of “disanalogies” and “monster analogies” that present difficulties for analysis.	Critique and refinement of arguments, including those justifying computational models of analogy.
Trans-hoc (*across this*) analogy	Analogizing as a continuing process, with unclear beginnings and endings.	Ongoing comprehension and understanding, which can occur in any of the other five functional contexts.

multiple-choice format. This permitted the investigation of how the participants reason. But how did the researchers reason in order to come up with the puzzles in the first place? Is that not also a form of analogical reasoning?

This realization—that analogy has varieties and functional forms—suggested a way to integrate a set of ideas that had remained disconnected in the analogy literature. A number of functional categories are listed in Table 3. In the following sections of this article we discuss each of these in turn.

AD-HOC ANALOGY: REASONING WITH THE GOAL OF CREATING ANALOGIES

Ad-hoc (to this) analogies occur in contexts in which the goal is to construct analogies de novo, typically using the explicit A : B :: C : D format, to meet certain criteria or to suit particular purposes. For example, if one wants to create a standardized intelligence test, one might want to construct three-word verbal analogies that are not too abstruse and that can appear with a number of alternative answers, only one of which is “correct.” A hypothetical macrocognitive model for ad-hoc analogy would incorporate the sorts of factors laid out in Table 4.

Ad-hoc analogies can be created so as to rely on essentially any form of relation: location (Pharmacy : Drugs :: Grocery : Food), function (Kitchen : Eat :: Bedroom : Sleep), subset (Robin : Bird :: Thoroughbred : Horse), dimensions (Deep : Shallow :: Expensive : Inexpensive), transformations (Seed : Tree :: Egg : Bird), devices and measures (Thermometer : Temperature :: Clock : Time), prevention (Dam : Flood :: Vaccination : Disease), illegal absence (Student : Truant :: Soldier :AWOL). These can be presented in the form of A : B: :: C : ?, with the alternatives selected so as to range anywhere between obvious and abstruse.

TABLE 4
Factors and Functions Involved in Ad-Hoc Analogy

Factor	*Function*
Goal	Analogies are sought to suit a particular purpose, involving their use as analogies.
Conceptualization	The purpose delimits (a) the form of the analogies (e.g., geometrical, verbal, etc.); (b) the categories or general domains for the terms that are to enter into the analogies, (c) the sorts of relations or transformations to be involved in the analogies; and (d) lists of terms that fall into the categories.
Conjecture	Analogies are constructed using more than four terms, since the analogy will explicitly have three, and the remaining terms can appear as alternative completions, only one of which is “correct” (i.e., it satisfies the intended relation).
Criticism	The analogies are then applied in their intended use. Their usefulness is somehow evaluated, according to measures or criteria that also stem from the initial purpose.
Cycle	Conjecture and criticism can be iterated until the goal is satisfied.

Having created a great many clever and frustrating analogies ourselves, it is clear that ad-hoc analogy requires mental effort and lots of trial and error, but there is more to it in terms of cognition, motivation, and affect. In running through this process of deliberately creating tricky analogies to meet certain criteria, one cycles through conjectures and criticisms, sometimes entirely abandoning one’s path. Many people have engaged in the sort of reasoning described in Table 4, including all the people who make analogy items for intelligence tests. But as far as we can tell, there is no research on how it is that people accomplish this—how they make analogies “out of the blue”—and there is limited theoretical extension to this in psychological theory and computer modeling.

PRE-HOC ANALOGY: REASONING WITH THE GOAL OF SENSEMAKING

Pre-hoc (before this) analogies occur in creative discovery contexts, as in most scientific analogy. In Thagard’s (1989) terms, the goal is to find coherent explanations, that is, to help one understand some given new concept or observation (and generate ideas for further hypothesis tests or observations). Gentner (1982) referred to a distinction between “explanatory” and “expressive” analogies. She argued that explanatory analogies are specific, clear, and systematic, whereas expressive (or poetic) analogies are rich and idiosyncratic (p. 127). More recently, Lee and Holyoak (2008) demonstrated that the formation of causal mental models can drive the processes of realizing analogies and making inferences from them.

For pre-hoc analogies, the goal and the “given” information differ from those in ad-hoc analogy. For pre-hoc analogy, all one really has at the start is the ill-defined topic concept—the thing to be explained or explanandum—as in questions like, “What are atoms?” or “What is heat?” Starting with such questions, one explores (and rejects) numerous alternative explananda (the vehicle terms of metaphors or the C terms in four-part analogies) such as “solar systems” and “fluids.”

A number of researchers have proposed an “encoding-retrieval-mapping” model (Holland, Holyoak, Nisbett, & Thagard, 1986; Holyoak & Barnden, 1994a,b; Thagard et al., 1990). For instance, Winston (1978, 1979) studied the use of analogy in scientific texts and problem-solving episodes (e.g., the use of hydraulic metaphors for electric circuits can lead students to "discover" Ohm's Law) and proposed a set of stages for what we are calling the pre-hoc context:

1. Recall or identification of a situation (C term) that might be used to understand the current problem (A term).
2. Testing the hypothesis by searching for a mapping of causal relations.
3. Generating a list or properties or relations that could apply (B and D terms).

In Klein and Weitzenfeld's (1982; Klein, 1987) study of avionics engineers, the problem-solving protocols revealed the sorts of stages described in Winston's model, with the same emphasis on the mapping of cause–effect relationships.

Holland, Holyoak, Nisbett, and Thagard (1986, chap. 10; Holyoak et al., 1994) proposed a similar model for the stages in creating analogies: encoding of the given term, retrieval of potential explanatory domains and explananda, a mapping operation (transferring knowledge or deriving inferences) yielding new rules or relations, evaluation of the inferences in terms of prior constraints, and the expression of generalizations based on the selected mapping.

In all such models, the first step—retrieval— is where the miracle happens. In an earlier era of psychology, this was called "The Höffding problem," after experimental psychologist Harald Höffding. He noted a problem, referred to as a paradox, of the relation of memory and recognition. How do you retrieve the "right" memory, one that has explanatory potential, unless you have already retrieved it from memory? Models of analogy that construct mappings based on the matching of preordained lists of semantic features have trouble with problem solving situations in which there are unknown or ill-defined factors. There might be some constraint-based mechanism for retrieving multiple candidate vehicles (Thagard et al., 1990), but this just pushes the miracle a step forward. Another answer to the Höffding problem is that this process sometimes really is a fishing expedition. Clement (1982) studied the spontaneous generation and use of analogies in the problem solving of physicists (mechanics problems involving springs, weights, etc.). In concert with the models of Winston and of Holyoak et al., the analogies did not lead to instant solutions but to a number of candidate solutions, which would then be analyzed and either confirmed or rejected.

Klein and Weitzenfeld, Clement, and Holyoak et al. agree that analogical problem solving (in the pre-hoc context) is not a single causal chain or fixed series of stages (a microcognitive view) but a set of mutually interdependent operations that act more like a cycle or parallelism of imagery–conjecture–criticism (a macrocognitive view) (see Klein et al., 2003). To paraphrase Holyoak (1984, p. 224), creative analogical problem solving seems less serial than the practiced solving of prepared analogical problems (which we call post-hoc analogy) (as in Mulholland et al., 1980). Factors and functions for pre-hoc analogy are presented in Table 5.

A major gap in computational modeling is that models of analogy are typically not integrated with models of problem solving (Holyoak, 2008a). Models for pre-hoc analogy must include criteria for when to reject candidate analogies due to explanatory breakdown, some sort of justification or evaluation processes (Falkenhainer et al., 1990; Kedar-Cabelli, 1988; Winston, 1979), and some process of special "radical restructuring" (Chalmers et al., 1992; Fass & Wilks, 1983). A process of restructuring the representation of concepts is generally regarded as being critical to analogy, and there are ample demonstrations of how analogical

TABLE 5
Factors and Functions Involved in Pre-Hoc Analogy

Factor	*Function*
Goal	The goals are creative discovery and the attempt to understand and explain "The Unknown" in terms of "The Known."
Conceptualization	The reasoner starts with a topic concept—term or terms that denote a to-be-explained phenomenon.
	The reasoner ponders whatever is known or believed about the concepts, functions, properties, that seem important to the phenomenon.
Conjecture	Through ruminations both deliberative and opportunistic, the reasoner somehow "finds" candidate vehicle concepts/terms, which bring with them their own domain.
	Comparisons are made between the vehicle and topic, and their domains.
Criticism	Comparisons are analyzed according to criteria that might include consistency, elegance, coherence, generativity, or other criteria.
	Candidate solutions can be refined or adopted for one or another purpose (e.g., the generation of research), or rejected.
Cycle	Conjecture and criticism can be iterated until the goal is satisfied.

inferences change the representation of concepts (Blanchette & Dunbar, 2002). Alternatively, computer models can embrace the possibility of representing ambiguity and conflict, and not merely taking ambiguity or conflict as an immediate command for resolution, as in the system of Fass and Wilks (1983).

POST-HOC ANALOGY: PUZZLE SOLVING OF INCOMPLETE ANALOGIES

Both ad-hoc and pre-hoc analogy involve the generation and use of analogies. A next step is to develop an explicit theory of how people comprehend and reason about the analogies that have been created by someone else, or how computers could be made to comprehend and reason about such analogies. *Post-hoc* (after this) analogies are ones that are given, perhaps already fleshed-out to some extent, and yet are waiting for explication or further analysis. One can attempt to discern "the" meaning of a post-hoc analogy, defined in terms of the intention or world knowledge of a human interpreter, or the semantics and inference processes within an AI system. The goal is to reveal the process of comprehending preformulated analogies or to suggest ways of forging computational models of the process of comprehending preformulated (as possibly incomplete) analogies, as in A : B :: C : ? Assumptions are made about the semantic base and the processes of inference constraint, and these are then expressed as a theory or implemented in a computational model.

Psychological theories deal primarily with the post-hoc analogy situation. An exemplar is the research and theory of Sternberg (1977a,b), based on his ambitious program of reaction-time research. In his experiments, participants were presented with the sorts of analogies that appear in intelligence tests, including pictorial and geometric analogies. In each trial, a participant would first be given a cue, such as the A : B pair. When the cue was understood, the participant pressed a button to go on. Next the participant was shown the completed analogy (A : B :: C : D), and the participant pressed one of two keys indicating "correct" or "incorrect." Control conditions included the withholding of a cue, the presentation of the A term only as a cue, or the presentation of A : B :: C: ? as the cue. In a number of detailed alternative mathematical models, Sternberg assumed that each operation (e.g., encoding, inference, etc.) was either present or not, and assumed that when an operation was present it took a fixed amount of time. An additive method could then be used to partial out component reaction times from the two sets of reaction time measures, for example, the estimated mean duration, for verbal and pictorial analogies, of the process of inference fell into the range of 130 to 300 msec, and that for mapping fell into the range of 200 to 250 msec.

For the data analysis, Sternberg assumed that the comprehension of analogy involves some variation on one or two basic sequences of mental operations, reflecting the strategies people use when adapted to this type of task and materials. One strategy is to go from a three-term analogy to an apperception of the underlying rule or relation, and then to selection of the one alternative completion that satisfies the rule. Another strategy is to work backward from the given alternative completions to discern the relation on which the analogy is based (see Evans, 1968; Mulholland et al., 1980; Pellegrino & Glaser, 1979).

An example of a computer system for post-hoc analogy is that of Martin (1990). Stages of processing in his system are:

1. Accept an input statement (e.g., "John gave Mary a cold") and express it in terms of the system representation (i.e., propositions in the form of semantic networks).
2. Establish the core meaning of the targeted metaphorically-used word (i.e., the network for "give") and the source concept (i.e., "cold").
3. Look up any concepts that the target and source share (e.g., "give" is related to "have," and one can "have" a cold).
4. Look up any stored metaphors using these concepts (e.g., to "give" permission, to "have" an idea).
5. Accept the use of "give" for use with the source concept (i.e., let the word "infection" fill the slot for "giving-result" in the network for the word "giving").
6. Extend the structure for the source (i.e., "having" a cold as a completion for the slot INFECT-RESULT).

Table 6 presents the factors and functions involved in post-hoc analogy.

TABLE 6
Factors and Functions Involved in Post-Hoc Analogy

Factor	*Function*
Goal	The goal is to reveal the process of comprehending preformulated analogies or to suggest ways of forging computational models of the process of comprehending preformulated (as possibly incomplete) analogies, as in A : B :: C : ?
Conceptualizationt	Pre-hoc analogies are analyzed in terms of their meaning.
Conjecture	The relation or transformation that relates the A and B terms is inferred. The relation is mapped, generating candidate D terms.
Criticism	A justification stage evaluates the alternative D terms.
Cycle	Assumptions are made about the semantic base and the processes of inference constraint, and these can be expressed as a theory or implemented in a computational model.

PRO-HOC ANALOGY: REASONING WITH THE GOAL OF UNDERSTANDING THE CONCEPT OF ANALOGY

This form is subtle but is nevertheless a form of analogical reasoning. *Pro-hoc* (for this) analogies are ones that are selected intentionally for the purpose of assessing or refining a theory or model of post-hoc analogy. Unlike ad-hoc analogy, in which a creator thinks about the content of particular analogies, in the pro-hoc context a theorist thinks about analogy as a concept and about theories or models of it. Both the goal and the “given” information fall at what might be called a meta-level. Pro-hoc analogy occurs whenever a post-hoc analysis of given analogies has yielded a psychological processing model, a stage theory, or a computational model that is then tested—it should predict something and should do so better than some alternative model. This is pro-hoc analogy at the level of the scientist’s actions.

Reasoning in the pro-hoc context also occurs every time a person thinks about analogy as a concept, but there is no empirical evidence about what happens in this situation. How experts (including cognitive scientists) reason about their metaphors and analogies should serve as an acid test for theories of analogical reasoning from cognitive science and for theories of reasoning from the psychology of science (Hoffman, 1992). Here is an opportunity to investigate analogical reasoning in the pro-hoc context. How do people (in general) reason when they are asked to explain what analogy is? How do they reason when they are asked to justify an analogy? What do people, of various ages and backgrounds think analogy is? Do they believe they reason using analogy? How do they (say they) reason when relying on analogy? Do they remember any analogies they have encountered? What processes are involved in the recall and recognition of analogies (as opposed to the recall of information that has been learned via analogy)? These are the sorts of interesting empirical questions that stem from a naturalist investigation.

Unlike Thagard’s (1988) computational model of “explanatory coherence,” most AI models are situated primarily in the pro-hoc analogy context. That is, the models first perform a post-hoc analysis, fleshing out a given analogy, and then they conduct a pro-hoc evaluation or justification stage. Such an AI system should yield satisfactory or sufficient structural analyses for test case analogies. Furthermore, pro-hoc analogy in AI includes reprototyping—the initial model (post-hoc analogy) is not only tested on the basis of new cases, but it can be refined, compared with alternative algorithms, and so on.

We can illustrate this by discussing the structure mapping engine (SME; Falkenhainer et al., 1990; Skorstad, Falkenhainer, & Gentner, 1987). The SME is especially relevant to this discussion because it has been based partly on results from psychological research. Although the SME has been applied in the analysis of pictorial and geometrical analogies (Gentner & Markman, 1993; Markman & Gentner, 1993), and has been applied as a general inference engine, it is primarily a model of verbal/conceptual analogy. The SME is a symbolic/logical system for analogy, as distinct from connectionist systems, even though some computational processes go on in parallel. What makes SME a symbolic system is that it operates by (a) algorithms that are dependent on syntax and (somewhat on) semantics (i.e., predication structures within propositions), and (b) inference axioms (rules, procedures) that are expressed in terms of propositions and the semantics of their constituent entities, predications, and parameters. Many representation schemes used in symbolic AI are all more or less like SME (frames with slots, ordered hierarchies or graphs, causal templates, etc.). (For a presentation of the details of the SME algorithm, see Forbus, Ferguson, & Gentner, 1994.)

Although the SME is not an emulation of a cognitive stage theory (e.g., it would not be used to predict comprehension reaction times), it has been tested for its fidelity as a means of building semantic representations and efficiently finding structural mappings in very large data sets. Furthermore, the SME is representative of most AI work in that it embraces many of the goals, core ideas, representation schemes, and processing mechanisms of other AI models. The SME stems from Gentner's (1983; Gentner & Jeziorski, 1993) analyses of metaphor in the history of science and from research on students' understanding of physical concepts. In this functional context, analogies are said to involve the mapping of domains—sets of entities and their interrelations, rather than just the mapping of individual entities and their relations.

The SME system emphasizes this mapping of relations—relations that hold among the nodes within the base domain are mapped to the relations that hold among the nodes within the target domain. Unlike the model of Holland et al. (1986), which does not perform a mapping process but instead constructs concepts, the model of Falkenhainer et al. (1990) has an explicit structure-mapping stage. Unlike the model of Carbonell (1983), which alters the base "until it fits" (and can therefore be used to generate plans for solving problems in the base domain), the system of Falkenhainer et al. processes analogies without making structural changes in the semantics of the target domain. Operations of the engine involve three general stages, reminiscent of theories of analogy we have already mentioned and discussed:

1. Retrieval of a domain for comparison, based on similarity (one-to-one mapping of relations) to the target domain. In this system, featural attributes can be ignored unless they play a role in the structure of concepts.
2. Mapping (of candidate inferences).
3. Evaluation of mapping quality in terms of a numerical analysis of similarities and differences—more similarities mean there's more "structure" in the mapping (e.g., if entries in the base and target have the same functions, this is evidence in favor of the mapping).

In the SME, constraints on inference can include the goals of the current problem-solving context, as in the model of Holland et al. (1986). On the other hand, the SME generates all possible relational mappings of source and target. Restriction comes not beforehand (e.g., by directed search) but "on the fly" through a process of selection of those that preserve most of the (canned-in) domain structure. And in a final process, an interpretation is "chosen" based on a numerical analysis that converges on maximal consistency. These two processes, directed search/selection and consistency evaluation, represent analogy in the pro-hoc context.

Gentner and her colleagues have successfully applied their model in the analysis of some scientific analogies (e.g., heat flow/water flow, atoms/solar systems). In one test case, the SME generated fewer spurious mappings than an alternative inference algorithm. In another test case using prosodic analogies, the engine's ratings of the "soundness" of alternative mappings matched the pattern of soundness judgments for human participants. The basic stages in the operations of the SME reflect the common post-hoc and pro-hoc spirit of most AI modeling efforts, expressed in Table 7. Of all the sets of factors and functions discussed so far (Tables 3–6), this one is the most stage-like, reflecting the structure of the processing models.

TABLE 7
Factors and Functions for Pro-Hoc Analogy, Typical of Many AI Models of Analogy

Factor	*Function*
Goal	The goal is to present test-case analogies to a model of cognition based on Post-hoc analysis.
Conceptualization	A test-case analogy (incomplete as far as the model is concerned) is given as input. The test case is understood in terms of the pre-existing semantic base (e.g., knowledge about "atoms").
Conjecture	A candidate vehicle concept and its domain (e.g., "solar systems") is selected. Relations are activated and transformations are enabled. A mapping is constructed in which possible relations and transformations are maintained in a conceptual graph, hierarchy, or tree structure.
Criticism	The mapping is evaluated according to some criteria (e.g., completeness, consistency, coherence, etc.).
Cycle	The evaluation enables a process of rerepresentation, remapping, and re-evaluation. This can be based on selection of an alternative candidate vehicle concept (and its domain).
Criticism	The alternative mappings are compared in terms of the evaluation criteria, and a "best" or a "better" choice is made. System choice performance can be compared to that of human problem solvers.
Cycle	Alternative algorithms are implemented at any of the preceding steps (i.e., the system can serve as a test bed or toolkit for assessing mapping algorithms).

CONTRA-HOC ANALOGY: POKING HOLES IN ARGUMENTS AND THEORIES

This type of analogy is related to pro-hoc analogy. The functional purpose of the latter is to support the creation of cognitive and computational models of reasoning. *Contra-hoc* (against this) analogy is intended to show the limitations of an argument, including the rationale for a computational model. To date, we can identify four distinct kinds of contra-hoc analogy: rebuttal analogy, disanalogy, misanalogy, and monster analogy.

Rebuttal Analogy

Rebuttal analogy is analogy used to persuade or to rebut an argument. Analogies are a commonly used rhetorical device (e.g., ways in which the recent U.S.–Iraq war is, and is not like the U.S.–Vietnam war). This functional form of analogy is distinct from explanatory analogy because the goal is pragmatic, that is, to affect attitudes or behavior. Rebuttal analogy would presumably be of interest in the fields of communication and rhetoric but has not been much researched apart from the study of the rhetorical functions of metaphors (Whaley & Holloway, 1996). Brian Whaley surveyed some rebuttal analogies found in the news reports and determined how rebuttal analogy is used to dispute someone's claims. In an example we encountered, a commentator on the U.S. government's No Child Left Behind education program quipped *"You don't fatten a pig by weighing it every day."* The clear message was that standardized testing was not a good approach to educational improvement.

> This simple yet elegant reasoning process is designed to have the instrumental effect of demonstrating that the fundamental principle is flawed in the opponent's argument. . . . A rebuttal analogy

> simplifies otherwise complex arguments into deceptively efficient retorts that rely on common sense and common experience for their reasoning power." (Whaley & Holloway, 1996, p. 164)

But rebuttal analogy often has a second purpose: to show by exaggeration that the opponent's argument is stupid and not just ridiculous. This is a ripe topic for research on the pragmatics and sociolinguistics.

A special instance of rebuttal analogy is a *counteranalogy*, that is, an analogy used to rebut an analogical argument (Shelley 2002a). For example, in a dispute with Isaac Newton, the German philosopher Gottfried Leibniz argued that God would not perform miracles. Being perfect, God would create a world in no need of intervention, just as a perfect watchmaker would create a watch in no need of repair. Newton's friend, Samuel Clarke, replied that God would certainly perform miracles, being a perfect governor of the world, just as a perfect king would oversee and intervene in the affairs of his kingdom (Shelley, 2003; 2004). In this instance, the counteranalogy is meant seriously; in other instances, it could be used to undermine the use of analogy in the original argument or a hypothetical argument. For example, in a discussion of the problems of computer viruses on the Internet, Stephen Savage of the University of California–San Diego said:

> A new network architecture isn't going to solve our problems. . . . The question I would like to ask people is, what are you going to do to the highway system to reduce crime? And when you put it that way, it sounds absolutely ridiculous, because while criminals do use the highway, no rational person is suggesting that if only we could change the transportation architecture that crime would go away. (Savage, 2009)

Disanalogy

A disanalogy is one that supports contradictory or mutually exclusive conclusions. In the history of scholarship on analogy, including analogy in science, it is often pointed out that analogies and metaphors often break down and are incomplete or inconsistent. We could use the example given earlier of atoms–solar systems, where it would be pointed out that the "orbiting" of electrons is a very different kind of relation from the "orbiting" of planets. In the history of science, it is often the ways in which analogies or metaphors break down that show a path toward refining a theory or conducting new experimental tests (Hoffman, 1980). Disanalogy goes beyond the idea that analogies are always incomplete or limited, to the idea that a single analogy can generate contradictory arguments. "In general, cognitive models of analogy have been built on the assumption that incompatible but causally supported inferences do not follow from a single analogy" (Shelley, 2002b, p. 82).

Here is an example we adapt from a recent televised discussion among some pundits.

> Commentator 1: The American economy has been a strong producer for two hundred years. The last thing you want to do in this economy is kill the goose that laid the golden eggs.

In context, the implication was that the United States should not create more regulations or "socialize" the banking system.

> Commentator 2: Yes, but the goose is sick and so you have to intervene somehow.

In context, the implication was that some additional controls are needed.

As another example, we can pursue the atom–solar system analogy. The counterargument is that the "orbiting" of electrons is a very different kind of relation from the "orbiting" of planets. It turns out that electrons in highly energized atoms act as “wave packets” that manifest an oscillating ionization pattern implying the existence of an ensemble of classical elliptical orbits—that is, they have aphelia and perihelia. There has been the suggestion that the addition of a radio frequency field could further limit the packets to a single elliptical path (Nauenberg, Stroud, & Yeazell, 1993). So the “orbiting” relation may be an actual limiting case rather than a “superficial” or “literal” relation.

Cameron Shelley (2002a,b) performed a formal analysis (using the representational schemes of Holyoak & Thagard [1989a,b] and Gentner [1983]) of a number of philosophical dialogs, including one from the writings of David Hume (e.g., God as builder of the world :: Man as builder of a house) and one from geology (the impact of Krakatoa’s dust on the environment :: the impact of asteroid impact dust on dinosaur extinction). Shelley showed that disanalogy involves adding into an analogy one or more additional higher-order (relational) predicates, resulting in a conclusion that is contradictory to a conclusion made from the original analogy. For the asteroid example, the counterargument was that volcanic eruptions other than Krakatoa, eruptions that approximated the estimated asteroid impact magnitude, did not result in mass extinctions. The new relational structure that was added to the analogy involved the estimation of the duration of the darkness that would follow an impact (or an eruption), with all of the other structural relations of the original analogy left unchanged.

Like rebuttal analogy, disanalogy plays its role in argumentation and disputation. Both forms of analogy are clever, but disanalogy is perhaps more so. Because the disanalog-based rebuttal is a variation on the source analogy, the proponent of the original analogy is closed off from objecting. We use the asteroid example to illustrate this and at the same time concretize the point that incompleteness, limitations, or inconsistencies are a strength, not a weakness of scientific analogies:

> Now consider the response of Alvarez et al. (1981) to Kent’s (1981) disanalogy to the asteroid impact hypothesis. Instead of trying to disallow the disanalogy, Alvarez et al. sought to reconcile its apparently incoherent conclusions. They noted that an appreciable difference in magnitudes remained between the Toba eruption and the asteroid impact. They introduced a new claim that a lower limit on extinction-producing events exists such that the asteroid impact exceeded this limit whereas the Toba eruption fell short of it . . . thus disarming the disanalogy. The lower limit is an empirical claim susceptible of testing. . . . Subsequent research has shown that there does appear to be such a lower bound on extinction-producing events. . . . So Kent’s disanalogy was not unfruitful: It extracted an additional claim from Alvarez et al. that led to a progressive program of research. (Shelley, 2002b, p. 94)

In looking at these kinds of analogies and comparing them to those that inhabit the literatures of cognition and computational science, Shelley (2004) and Veale and Keane (1997) suggested that cognitive and computational models of analogy have been biased by the focus on a certain form of analogy and selection of certain kinds of cases. An additional form of contra-hoc analogy underscores this point.

Misanalogy

Another form of rebuttal analogy discussed by Shelley (2002a) is the misanalogy. A misanalogy is an analogy that has been corrected in light of novel information. For example, Alvarez,

Alvarez, Asaro, & Michel (1980) argued that an asteroid impact would kick debris into the air that would block out the sun for perhaps three years. Kyte, Zhou, and Wasson (1980), in response, argued that the debris would remain airborne for only a few weeks. The difference is crucial because Alvarez et al. believed that a long blackout was needed to cause the global dinosaur extinction. Kyte et al.'s correction, if accepted, would undermine this conclusion. As with disanalogies, the occurrence of misanalogies emphasizes the defeasibility of analogical inference. That is, conclusions based on analogies may be subject to revision as investigators encounter new information.

Misanalogy, as a natural kind of analogizing, blends into "continuous analogy," which we discuss in the next main section. First, however, there are more forms of contra-hoc analogy to be presented.

False Analogy

Related to the misanalogy is the *false analogy*. A false analogy results from revising an analogy in the light of novel information; however, the result is simply the destruction of the original analogy. Shelley (2003) discusses an analogy that was given, tongue-in-cheek, by the philosopher David Hume:

> A patricide is in the same relation to his father as a young oak to the parent tree, which, springing up from an acorn dropped by the parent, grows up and overturns it. We may search as we like, but we shall find no vice in this event. Therefore there can be none in the other where the relations involved are just the same. (Hume, 1739–1740/1978, p. 467)

On the surface, we seem to have analogy that implies that patricide is no crime. However, the analogy itself does not stand up to scrutiny. The notion of moral responsibility does not apply to the oak tree, as Hume says, but it does apply to the patricide because he is a human being. Since moral responsibility is represented on one side of the analogy but not the other, the result is a failure of structural consistency. In brief, the analogy is really not an analogy after all.

Monster Analogy

The concept of the monster analogy (Hoffman, 1995) was introduced precisely to make the point that analogy can involve multiple formats and a boundless variety of semantic and conceptual relations. More specifically, monster analogies were intended to serve as grounds for disputation of the assumptions and postulates underlying cognitive and computational models of analogy. This functional type of analogy puts us right up against the problems of creativity, "syntactic infinity" (Katz & Fodor, 1963), and "semantic infinity" (Hoffman & Honeck, 1976). Table 8 presents some examples of monster analogies.

Monster analogies were defined (somewhat whimsically, in Hoffman, 1995) as analogies in which: (1) Creating a good one can drive the creator nuts, and either (2) no existing computer model approach can accept it as input without relying on redescription based on some sort of ad hoc criterion, or (3) no existing algorithm can even come close to being able to solve it in the ways humans can. Table 9 presents a tentative analysis of the factors and functions for Monster analogy.

TABLE 8
Some Example "Monster" Analogies

Analogy	*Explanation*
Plato's analogies: Philosophy :: Maxwell's analogies : Physics	Analogy can be about analogy.
Dog : Hound :: Bother : Type	If the first two terms are polysemous they can have their own explanations in the second two terms. In this example, to "dog" someone is to "hound" (bother) them, with hound also being a type of dog.
River : Story :: Milk : ? Horse : Time :: Stone : (a) king, (b) book, (c) girl, (d) train	Such anomalous analogies can be solved, if people are given enough semantic rope (M. G. Johnson, 1975)
Beef : Stake :: Hand : Brake Helicopter : Hummingbird :: Submarine : ?	This first example of a garden-path analogy might be solved on the basis of common phrases, but the solution might just as well be the rhyming relation. In theory, for any analogy having more than one solution, it is possible to construct a higher-order relation that embraces the individual solutions. The second example could be based on machines versus life forms that fly or swim, but it could be based on the idea of things capable of station-keeping. The word "seahorse" would satisfy both interpretations.
Drag : Pull :: Travel : Plow	Some items in the Miller Analogies Test rely on apparent anomaly. In garden-path anomalous analogies, the anomaly is only apparent. In this example, to drag is to pull something, but how does this map to the C and D terms? (Hint: Types of contests vs. ordinary functions.)
Toast : Food :: Toast : Honorific Snow : Video :: Snow : Traffic	Analogies can rely on polysemy and have four terms but only three words. These examples are a "type-of" relation.
Field : Mouse :: Prairie : Dog versus Dog : Mouse : Field : Prairie Mount : Horse :: Board : Plane versus Mount : Board :: Horse : Plane	Changes in the ordering of terms can result in nontrivial change in the relations. The first example of a "duplex analogy" involves types of rodent versus a domesticated–undomesticated relation. The second example involves means of getting onto a mode of transportation versus tools for advertising or for carpentry.
Bird : Sparrow :: Mammal :: ? Bird : Sparrow :: Mammal : Dog : Pig: Rabbit	Multiterm analogies take the linguists' notion of inflectional analogy into the semantic realm.
Copernicus, Brahe, Galileo : Astronomy :: Bohr, de Broglie, Heisenberg : Quantum mechanics The odorless child inspected the chocolate audience : Semantic anomaly :: Boy book read the : Syntactic anomaly	There is no a priori reason why a term in an analogy has to be a single word. A term might even be a sentence.
This analogy : Ill-formed syntax :: Ill-formed semantics : Horse. "This analogy refers to itself" : Self-reference :: "This analogy does not refer to itself" : Contradiction	Analogies can be self-referential and ill-formed. Because the analogy is correct in asserting its own ill-formedness, does it really have an anomalous semantics? The second example presents a similar conundrum.
(Feathers : Birds :: Hair : Mammals) : Simple :: This analogy : Complex	Analogies can be embedded inside other analogies.

TABLE 9
Factors and Functions for Monster Analogies

Factor	*Function*
Goal	The goal is to present test-case analogies to a cognitive or computational model of analogy based on post-hoc analysis.
Conceptualization	An analogy that is either a monster or an analogy that deviates from the four-term format is given as input. The test case is understood in terms of the pre-existing semantic base.
Conjecture	Either the input is of a form that cannot be accepted as input (constituting a QED test), or a candidate solution is generated.
Criticism	The solution is evaluated according to criteria grounded in human semantic and syntactic flexibility. Evaluation may hinge on the fact that humans can create multiple acceptable solutions, and higher-order solutions that embrace sets of solutions. The evaluation is likely to culminate in a determination that the computational model is incomplete in one or more ways.
Cycle	Alternative algorithms or additional computational techniques are implemented.

TRANS-HOC ANALOGY: ANALOGIZING AS A CONTINUOUS PROCESS

Cognitive theories of analogical reasoning are dynamic insofar as they posit hypotheses about mental processes that are placed into a causal sequence. Computer models of analogical reasoning are dynamic in that they actually *do* things (e.g., construct mappings). But there is an important sense in which neither the theories nor the models are dynamical. A person may at one time perceive a resemblance or distinction between *x* and *y*, and then go on to other thoughts about other things that he or she is reaching to understand. At a later time they may return to *x* and *y* and reject the resemblance or distinction for any of a variety of reasons, and then at a still later time come back and realize that the distinction or resemblance falls at some other level or that there is some other resemblance or distinction. Or not.

In *multiple analogies*, several source analogs are mapped to a target (Shelley, 2003) and evaluated but not necessarily with the immediate purpose of rejecting any of them to result in a single best choice. This form of analogizing occurs in contexts in which a single analogy (an analogy with one source) does not satisfy the analogizer’s goals. One of the main challenges of multiple analogies is to wrap a group of analogies into a coherent whole, an umbrella analogy that tells a single, understandable story and supports the conclusion arrived at. Multiple analogies work differently than single analogies and so cannot be modeled appropriately in a computational framework designed expressly for single analogies (Shelley, 2003).

Furthermore, it is not *always* the case that there is a clear-cut single point in time at which one can definitively say that a process called analogical reasoning has been engaged. Likewise at the other end: rebuttal analogy, misanalogy, false analogy, and disanalogy are clearly extended over time. If the study of analogy and metaphor in the history of science has taught us anything, it is that metaphors and analogies are not always either accepted or rejected once and for all. Indeed, most of the time they continue to be debated, refined, screamed at for decades, even in the musings of their originators. So, it is not *always* the case that there is a clear-cut ending to a process we call analogical reasoning.

If this is so, why are theorists and theories constrained to think about analogical reasoning as a process that always has clearly specifiable beginnings and endings? Why must cognitive

TABLE 10
Factors or Functions Involved in Trans-Hoc Analogy

Factor	*Function*
Goal	Any of the other functional contexts—ad-hoc, pre-hoc, post-hoc, pro-hoc, or contra-hoc
Conceptualization	One attempts to specify whatever is known or believed about the concepts, functions, properties, that seem important to the phenomenon one wishes to understand or the concept one wishes to create.
Conjecture	One perceives resemblances or distinctions between the phenomenon and other phenomena.
	The resemblances or distinctions are evaluated, refined, and possibly rejected.
Cycle	Conceptualization–conjecture can be repeated.
	Reasoning may be interrupted for any of a variety of reasons and for any length of time.
Criticism	Candidate understandings or concepts are tentatively adopted, that is, put to some use.
Conjecture	The understandings or concepts undergo even further refinement even though they might have been put to some use.
Cycle	The preceding can be repeated and may be interrupted for any of a variety of reasons and for any length of time.

theories end with some sort of determination that the analogy is "understood" or is "coherent?" Why must models end with the choice of a single mapping or the evaluation of a mapping?

Trans-hoc analogy is continuous or continuing analogical reasoning, performed in service of the drive for comprehension. Actually, it is perhaps the most common of all the forms of analogy. And yet, it requires a rather different sort of theory from all other forms of analogical reasoning. In addition, any of the other goal-related forms (ad-hoc, pro-hoc, contra-hoc, etc.) can be manifested as the trans-hoc process (e.g., perceiving, rejecting, and refining analogies while reaching for analogies to use in intelligence tests). Factors or functions involved in trans-hoc analogy are presented in Table 10.

IMPLICATIONS

We have discussed some functionally distinct kinds of analogizing. These mandate differing treatments in psycholinguistics, sociolinguistics, and computational modeling because they involve

- Differing goals.
- Differing kinds of "given" information.
- Differing kinds of constraint.
- Differing styles of justification and criticism.
- Differing purposes, spanning pragmatics or rhetoric, and problem solving.

For computational modeling, each involves differing kinds of sequences of operations if one chooses to think in terms of microcognition and stage theories—or differing kinds of cycles and parallelisms if one prefers to take a macrocognitive perspective. For pre-hoc and pro-hoc analogies, one may rely on systems that have been based on formal post-hoc analysis, but for a post-hoc analysis to be applied in the pre-hoc or pro-hoc context there must be additional algorithms for searching for candidate completions, in addition to the algorithms for processing proposed completions to assess them for such things as coherence or completeness.

This naturalistic investigation highlights the fact that only certain aspects of analogical reasoning have come under scrutiny and have been computationally modeled in psychological research. A number of possible lines of research remain wide open. We now present our conclusions regarding each of the major themes of this article.

Forms and Formats of Analogy

Analogy theories and models generally assume that analogy takes the form A : B :: C : D. But there is clear historical precedent for the claim that this is restrictive. When the notion of analogy was carried over from geometrical to linguistic forms, it was used to capture all manner of linguistic relations. For instance, one could refer to "oratorem : orator :: honorem : honor" as a "proportion" (de Saussure, 1916/1959, p. 161). Here is another example:

> Scream : Screams : Screaming : Screamer : Screamed :: Dream : Dreams : Dreaming : Dreamer : Dreamed

As another example of *n*-term forms, of the five noun declensions in Old English, only the masculine strong declension involved using "s" for pluralization, but by the Middle English period all declensions had adopted (by a hypothetical process of analogy) the "s" pluralization. In this way, declensions were understood as having developed by analogy. Grammarians also used analogy as an explanation of historical change of word forms and tense inflections (similar meanings should be represented by similar forms). The change of verbs from strong to weak, and even the creation of new words ("kingdom" and "duke" gave rise to "dukedom") were also described as change by analogy. In using analogy to explain language change, linguists of the 18th and 19th centuries were limited only by the number of languages and the number of inflections being compared (see Bloomfield, 1933). To grammarian Nicholas Beauzee (1767), theoretical psychologist Wilhelm von Humboldt (1836/1960), and linguist Benjamin Wheeler (Wheeler, 1887), analogy governs all human language at the level of syntax and case relations. Analogy was regarded as a basic aspect of all language, including phonetic and syntactic change, and yet nowhere was there an assumption that analogy always involves four and only four terms.

Would anyone expect a computer system for computing verbal analogies to be able to process geometrical analogies, or a system for letter-string analogies to compute analogies that mix formats? Would anyone expect a system for solving verbal analogies to solve analogies that mix words and geometrical forms? Advances by such pioneers as Keith Holyoak, Ken Forbus, Dedre Gentner, Paul Thagard, and others have included efforts to achieve such machine capabilities. That being said, it is by no means obvious that a system for computing analogies must, should, or even could deal with analogies in all their various forms, functional contexts, and monstrous convolutions. Modelers often claim that their systems are general, perhaps a general inference engine based on constraint satisfaction, for example, but this claim is based on the analysis of a single type of analogy that happens to be especially conducive to the kinds of structural analysis that are engaged (Russell, 1992). As long as one has confined the scope of a theory or a computational model to a particular form of analogy, then it can be too easy to find evidence for serial processing, for stage-like cycles of conjecture and refutation, and so forth (e.g., Mulholland et al., 1980).

Is Analogical Reasoning a Basic Mental Operation

Despite the bold claims, has there ever really been any evidence for the assertion that analogy is a somehow a singular or fundamental mental operation, let alone one that is innate or universal? Furthermore, the fact that the standard analogy format can be used to give meaningful or symbolic descriptions of comparisons is not sufficient to support the claims that:

1. *Analogy* is a neutral term for all types of comparison, including similarity.
2. Those relations that the theorist has described bear a necessary and strong resemblance to the relations or ideas someone else might experience.
3. A special process called analogical transfer underlies problem solving in general.

There Is More to It Than Similarity

The "recognition of similarity" is sometimes regarded as a basic principle underlying all inductive arguments, and not just analogy (Corbett, 1971; Gentner & Jeziorski, 1993; Hesse, 1966; Holland, Holyoak, Nisbett, & Thagard, 1986). To one of the prophets of automaton theory, Etienne de Condillac (1756/1971), analogy is any relation of similarity. Entire cognitive theories of analogy, laboratory research paradigms, and grand computational models of analogical inference are based on the singular notion that analogy, or analogical mapping, is based on this one relation of similarity (e.g., Markman & Gentner, 2000). "Similarity is a process that itself is a fundamental cognitive function" (Medin, Goldstone, & Gentner, 1993, p. 275). After all, unless something (a concept, a relation, or a structure of related concepts) can be seen as similar in some respects to some other something, then matching or mapping is not possible.

In the research paradigm for the study of analogical transfer, problem similarity can be superficial. For example, Reed, Ernst, and Banerji (1974) presented participants with two problems to solve. The first was the classic "Missionaries–Cannibals" problem: Three missionaries and three cannibals have to cross a river, but the boat can only carry two persons. If the missionaries on either bank are outnumbered by cannibals, they would be eaten. The task is to find a schedule of crossings that permits the missionaries to cross safely. The transfer problem was about three jealous husbands who, with wives, have to cross a river, but the boat can carry only two persons. Transfer indeed.

It is widely assumed that words, sentences, and other expressive forms are things that "have" meanings or semantic features, and that these possess something called "similarity." This sort of reification is manifest throughout the history of linguistics, psychology, and other disciplines (Hoffman, Cochran, & Nead, 1990; Reddy, 1979). By this view, the goal of analysis is often to express "the" meaning of a sentence or proposition or to express "the" similarity of two things. This structuralist–reificationist view has been carried over to the analysis of analogy.

The Reification of Analogy

Sacksteder (1979) asked the question of whether logic justifies analogy, or analogy justifies logic. He began by defining analogy as the postulation of a perceived similarity (rather than something justified by formal logic alone). His second premise was that analogy entails certain inferences based on the similarity of qualities, the similarity of relations, or the similarity of structures. The third premise was that the similarities define transformation rules that make arguments valid or plausible. One could conclude from these premises that analogy underlies literally all of logic—because all logic can be said to rely on structured inference. What has been done here is to define the structures of logic in such a way as to conform to the similarity theory of analogy. To paraphrase Sacksteder (1979, p. 35) and Shelley (2002b, p. 95), analogy is something we have created by formulating logic such that arrangements are both formed and ruled by the concept. This point deserves emphasis: Given that analogy (especially the four-term analogy used in mental tests) has become ingrained in Western civilization, it is now possible to claim that analogy is necessary for problem solving precisely because the concept of analogy and the analogy format were invented to label and describe exactly the sorts of phenomena that problem solving involves. One can even reverse the order and assert that "similarity is like analogy" (Gentner & Markman, 1995, p. 111). Saying that analogy is basic to logic or cognition is just like saying "This man behaves crazy because he is schizophrenic." It is a description, not an explanation.

Metaphor and Analogy

Reasoning by analogy is a set of skills acquired through experience, mostly with pro-hoc analogy puzzles, typically the of sorts that appear in pedagogical and assessment contexts. Metaphorical thinking, on the other hand, seems to be a pervasive, and perhaps fundamental feature of language, development, and cognition in general (Indurkhya, 1992, 1994; Lakoff & Johnson, 1980). There is no clear empirical evidence—there never really has been—for the premise that analogy underlies metaphor (see Gentner, Bowdle, Wolff, & Boronat, 2001) or that proverbs are analogies because they too reduce to relational similarity (Markman, Taylor, & Gentner, 2007; for an alternative view, see Honeck, 1997). The burden of proof falls on the shoulders of those—and there are many—who assert uncritically that metaphors are "merely" analogies.

The converse—that metaphor underlies analogy—has also been argued (e.g., Beck, 1978). The difference between metaphor and analogy in problem solving is certainly functional whether or not it is cognitive. Metaphor and imagery seem to lace freely through the stream of consciousness, whereas analogical reasoning is a set of beliefs, goals, attitudes, standard formatting practices, and skills that may or may not be applied in the exploration of any given metaphor. Most statements in the explicit analogy format represent the stage at which perceptions and relations have already been fleshed out according to the analyst's purposes and goals (Medin & Ortony, 1989).

The Mysteries of Macrocognition

As this discussion suggests, many bold claims serve as the bedrock of the literature (e.g., Bowdle & Gentner, 2005; Holyoak, 2008a,b; Markman & Gentner, 2000). For example:

- Both metaphor and proverbs can be sufficiently understood as being mappings based on relational similarity.
- Analogy is a single and fundamental mental operation, even innate and universal.
- A process called analogical transfer underlies problem solving in general.

- Similarity itself is a fundamental cognitive process.
- Linguistic forms “have” meanings or semantic features, and that when regarded pair-wise they possess a property called “similarity.”

Some of this is not disputable. Knowledge and processes of the type that are sometimes described using the analogy format often seem to play a causal role in learning and problem solving. Reasoning that can be described using the analogy concept is an important part of creative problem solving in general and scientific creativity in particular. However, the mental operations foundational to analogical thinking seem to include a number of mysteries of macrocognition:

1. The apperception of resemblances and distinctions.
2. The ability to create well-defined or analytical formats for laying out propositions that express meanings and perceptions.
3. Mental imagery.
4. Metaphor.
5. Semantic flexibility.
6. Inference constraint.

Analogy does not exist in the sense that it is clearly distinguishable from this set of cognitive processes, except in that it is a concatenation or derivative of this set cast in stone as the classic four-term analogy. Certainly analogy does not exist in the sense that it carries explanatory value without reference to these cognitive processes.

This view accords with some expositions on analogy that have appeared since the flurry of interest in the 1980s and 1990s. The pioneering researchers have seen value in regarding analogy as being an element in a more general theory of inference constraint (e.g., Forbus, 2001; Hofstadter, 2001; Thagard, 2000).

In general, AI models and psychological theories have as their goal the post-hoc analysis (i.e., “build me a theory”) of incomplete ad-hoc or pre-hoc (i.e., preformulated) verbal analogies so as to generate “solutions”—single-candidate completions (literal paraphrases) that satisfy certain pre-hoc (explanatory) and pro-hoc (i.e., justification) criteria. Broadening the scope of research, theory, and modeling beyond that situation may be helpful as cognitive science and AI grapple with the fundamental mysteries of macrocognition—mysteries that not only seem to be the underpinnings of so-called analogical reasoning but that are also linchpins in strong definitions of AI.

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