Analysis of short-pulse laser photon transport through tissues for optical tomography

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We describe a method for analyzing short-pulse laser propagation through tissues for the detection of tumors and inhomogeneities in tissues with the goal of developing a time-resolved optical tomography system. Traditional methods for analyzing photon transport in tissues usually involve the parabolic or diffusion approximation, which implies infinite speed of propagation of the optical signal. To overcome such limitations we calculate the transmitted and reflected intensity distributions, using the damped-wave hyperbolic P_1 and the discrete-ordinates methods, for a wide range of laser, tissue, and tumor parameters. The results are compared with the parabolic diffusion P_1 approximation. © 2002 Optical Society of America

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Time-resolved optical tomography is an example of diagnostic methods that use short-pulse laser interactions with highly scattering and absorbing media such as biological tissues and is of great scientific, engineering, and medical interest.[1–5] Short-pulse probing techniques have distinct advantages over conventional large-pulse-width or cw lasers, primarily because they can convey additional information about the tissue's interior by temporal variation of the observed signal. When cw laser sources are utilized, the information available is the magnitude of the net attenuation and the angular distribution of the transmitted or reflected signal. The distinct feature of a short-pulse laser is the multiple-scattering-induced temporal signature that persists for times greater than the duration of the source pulse and is a function of the source pulse width, the scattering and absorbing properties of the medium, and the location in the medium where the properties undergo changes.

Most previous studies considered the parabolic diffusion approximation,[1,3–6] which one derives from a complete transport equation by neglecting certain time-derivative terms in the radiative transport equation. Some of the studies cited have experimentally investigated short-pulse laser transport through tissues and indicated that the parabolic approximation is adequate only for thick tissue samples. Also, the results of these parabolic models do not match most of the available experimental results and cannot accurately account for the change in properties at internal interfaces.[2,7] A Monte Carlo simulation, which includes finite speed of propagation of radiation transport but entails great computational expense, was considered by many researchers.[4,8] The Monte Carlo results have been shown not to match the parabolic diffusion results for tissue samples of small thickness.[9] Some discussions of these limitations have been reported in the literature.[1,10] In addition, in most previous research, appropriate phase functions were not considered or a simplified form of a scattering phase function distribution was used to represent tissue, leading to inaccurate values of transmitted and reflected signals.

In this Letter we show that the temporal transmitted and reflected optical signals from the tissue samples obtained with the hyperbolic discrete ordinates method are significantly different from the commonly used parabolic diffusion approximation and hyperbolic P_1 models. For brevity, only the effects of variation of the laser pulse width, tumor properties and location, and scattering phase-function distribution on the transmitted and reflected signals are analyzed here by the hyperbolic discrete-ordinates method.

The physical case under consideration is a one-dimensional scattering and absorbing layered tissue medium of thickness L, infinite horizontal extent, and azimuthal symmetry. For illustration, we assume that inhomogeneities such as tumors that have different properties from those of the surrounding healthy tissues are present at depths from L_1 to $L_1 + L_2$ from the tissue surface (i.e., inhomogeneity thickness, L_2). We use the following radiative transfer equation to analyze short-pulse laser propagation through tissues[11]:

$$\frac{1}{c}\frac{\partial I(x,\mu,t)}{\partial t} + \mu\frac{\partial I(x,\mu,t)}{\partial x} = -\sigma_e I(x,\mu,t) + \frac{\sigma_s}{2}\int_{-1}^{1} I(x,\mu',t)p(\mu\rightarrow\mu')\mathrm{d}\mu' + S(x,\mu,t), \quad (1)$$

where I is the intensity [W m^{-2} sr^{-1}]; c is the speed of light in the medium (= speed of light in vacuum divided by the refractive index of the medium); x is the Cartesian distance; t is the time; σ is the radiative coefficient (e and s refer to extinction and scattering, respectively); μ is the cosine of θ, where θ is the polar angle measured from the positive x axis; p is the scattering phase function, and S is the source term. This is an integrodifferential equation in which the partial differentials represent a hyperbolic form of equation.

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The phase function is represented in terms of a series of Legendre polynomials P_m as

$$p(\Theta) = \sum_{m=0}^{M} a_m P_m(\cos\Theta), \tag{2}$$

where Θ is the scattering angle, M is the order of anisotropy, and a_m are the coefficients in the expansion.

The pulsed radiation that is incident upon the tissue medium is a square pulse with a temporal duration (or pulse width), t_p. The intensity in the medium can be separated into a collimated component, which corresponds to the incident source, and a scattered intensity. If the collimated intensity is I_c, then I is the remaining part, which can be described by Eq. (1). The collimated component of the intensity, I_c, is represented by

$$I_c(x,\mu,t) = I_0 \exp(-\sigma_t x)[H(t - x/c) - H(t - t_p - x/c)]\delta(\mu - 1), \tag{3}$$

where I_0 is the peak power at the surface, $H(t)$ is the Heaviside step function, and $\delta(t)$ is the Dirac delta function. Source function S for the scattered intensity field is then given by

$$S(x,\mu,t) = \frac{\sigma_s}{4\pi}\int_{-1}^{1} I_c(x,\mu',t)p(\mu' \to \mu)\mathrm{d}\mu'. \tag{4}$$

The equation of transfer, Eq. (1), is complicated because of the integral on the right-hand side, which corresponds to the in-scattering gain term. To reduce the integral to a simpler form, we use two techniques, namely, the linear spherical harmonics expansion (P_1) and the discrete-ordinates method.

Under the P_1 approximation the intensity is considered to be a linear function of direction cosine μ.[11,12] The resultant hyperbolic wave equation obtained by integration of Eq. (1) over all solid angles is given by[12]

$$\frac{3}{c^2}\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} + \frac{3}{c}(\sigma_a + \sigma_e - \sigma_s g)\frac{\partial u}{\partial t} + 3(\sigma_e - \sigma_s g)\sigma_a u = (\sigma_e - \sigma_s g)\frac{3}{2}\int_{-1}^{1} S\mathrm{d}\mu - \frac{3}{2}\int_{-1}^{1}\frac{\partial S}{\partial x}\mu\mathrm{d}\mu + \frac{3}{2}\int_{-1}^{1}\frac{1}{c}\frac{\partial S}{\partial t}\mathrm{d}\mu, \tag{5}$$

where u is the intensity averaged over all solid angles and g is an integrated phase function.[12] Equation (5) indicates that, whereas the propagation speed of the original laser pulse is c, the propagation speed along the x direction of the resultant hyperbolic wave of u is $c/\sqrt{3}$.

We obtain the parabolic form of the equation that is widely used in neutron transport[13] and that has now been adopted by researchers in optical tomography applications[14] by neglecting the first term on the left-hand side and last two terms on the right-hand side of Eq. (5). It is assumed that the absorption coefficient (σ_a) is negligible compared with $\sigma_e - \sigma_s g$. The resultant classic diffusion equation is given by[6]

$$\frac{1}{c}\frac{\partial u}{\partial t} - \frac{1}{3(1-g)\sigma_s + \sigma_a}\frac{\partial^2 u}{\partial x^2} + \sigma_a u = \frac{1}{4\pi}\int_{4\pi} S\mathrm{d}\Omega \tag{6}$$

Equation (6) implies an infinite speed of propagation of the optical signal.

In the hyperbolic discrete-ordinates method, the integral on the right-hand side of Eq. (1) is replaced by a quadrature of the Gaussian, Lobatto, or Chebyshev type. If x_i are the quadrature points between the limits of integration, -1 to 1, that correspond to a $2K$-order quadrature, and w_i are the corresponding weights, Eq. (1) is reduced to the following system of coupled hyperbolic partial differential equations:

$$\frac{1}{c}\frac{\partial I_i(x,t)}{\partial t} + \mu_i\frac{\partial I_i(x,t)}{\partial t} = -\sigma_e I_i(x,t) + \frac{\sigma_S}{2}\sum_{j=-K}^{K} w_j I_j(x,t)p(\mu_j \to \mu_i) + S(x,\mu_i,t), \qquad i,j \neq 0, \tag{7}$$

where $I_i(x,t) = I(x,\mu_i,t)$. The hyperbolic wave speed of I_i along the x direction that corresponds to the discrete ordinate μ_i has the magnitude $\mu_i c$.

We obtained the transmitted and reflected signals numerically by solving the transient radiative transport equation, using the hyperbolic discrete-ordinates method,[15] the hyperbolic P_1 approximation,[10] and the parabolic diffusion approximation.[10] The optical properties considered in this Letter are those of biological tissues and tumors.[1,10] A forward-peaked phase function is used to represent the tissue medium.[11] Figure 1 shows the simulated reflected signal from a tissue sample for three models. The magnitudes of the reflected signals for these models match only for long times. For shorter times, each model predicts a different temporal shape and magnitude of the reflected signals. The hyperbolic and parabolic P_1 models give an unrealistic negative reflected signal value at short times, as is evident from Fig. 1, and are clearly not appropriate models for thin tissue samples. The hyperbolic discrete-ordinates method, which is the most accurate one, is therefore used here. The average computational time is 20 s on a Pentium III, 600-MHz PC for a 12–discrete-ordinate Gaussian quadrature. The results of hyperbolic

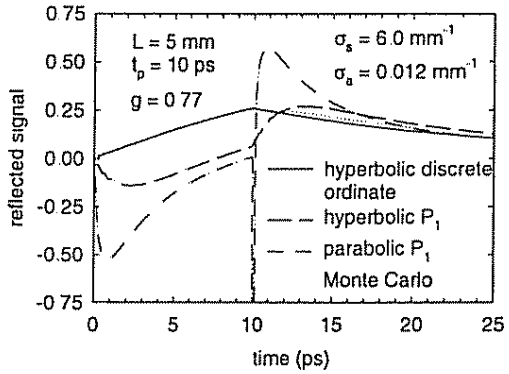

Fig. 1. Signal reflected from a tissue medium for the three models of tissue-analysis methods discussed here.

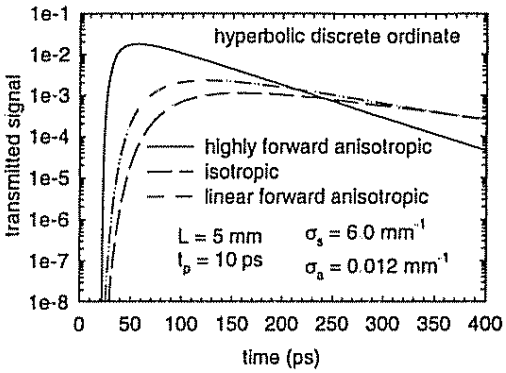

Fig. 2. Signal transmitted through a tissue medium for various phase-function distributions.

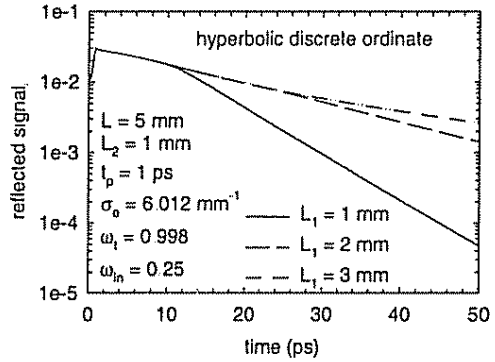

Fig. 3. Signal reflected from a multilayered medium for several inhomogeneity locations.

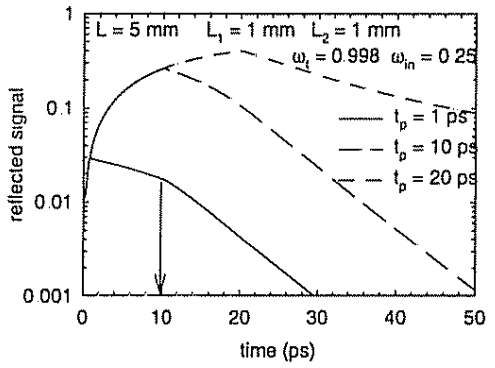

Fig. 4. Signal reflected from a multilayered medium for several laser pulse widths.

discrete-ordinates method match those of the Monte Carlo simulation.[11]

Figure 2 shows the effect of the phase-function distribution on the simulated transmitted signal. Most researchers employ a simple isotropic or linear forward anistropic phase-function distribution rather than the realistic forward-peaked anisotropic phase function distribution to represent tissue samples. One can see from Fig. 2 that the decay of the transmitted signal is much slower for isotropic and linear forward anisotropic phase functions than for the realistic highly forward anisotropic phase function. The photons tend to remain inside the medium for longer times for isotropic and linear anisotropic models and thus give incorrect transmitted signal values. It is evident from Fig. 2 that the values of the transmitted signals are zero until the exponentially decaying source pulse has traversed the medium at the speed of light.

The effect of the tumor's location relative to the tissue surface is shown in Fig. 3 for a multilayered medium; i.e., the tissue–inhomogeneity–tissue layer and the albedo ($=\sigma_s/\sigma_e$) of tissues (ω_t) and tumors (ω_{in}) are different. As soon as the laser pulse reaches the tissue–tumor interface, an inflection in the reflected signal is observed (Fig. 3). This effect is more pronounced for a tumor closer to the tissue surface, i.e., for smaller L_1. Figure 4 shows the effect of the laser pulse width on the reflected signal for a multilayered medium. It is evident from the figure that the shorter the laser pulse width is, the more clearly defined the interface between tissue and tumor will be. For 10- and 20-ps pulse-width lasers, the inflection point corresponds to the time when the laser pulse is shut off and not to the tissue–tumor interface, as in the case of 1 ps. For large-pulse-width laser sources, multiple-scattering effects will smear out the sharp inflection in the reflected signal. Thus it is observed that the temporal spread of the scattered signal can be correlated to the medium's characteristics. The significance of this study is that it will provide a guidance tool for the development of time-resolved optical tomography for biomedical imaging of tissues.

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